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RESOLUTION INTO SERIES OF THE THIRD BAND OF THE CARBON BAND-SPECTRUM.

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SINCE publishing my preliminary studies on the law of spectral series in this JOURNAL, 6, 65, 1897,[1] I have made many trials of the formula

$$\lambda = \frac{p_0 + \quad . \quad . \quad . \quad . \quad + p_r (n+c)^r}{q_0 + \quad . \quad . \quad . \quad . \quad + q_r (n+c)^r} \tag{1}$$

and must now repeat, with a mixture of regret and satisfaction, that it is the only formula I can recommend.

I can say in its favor that it has always given the expected rapid convergence of both series in the numerator as well as in the denominator. While in simple cases Professor Pickering's formula ($r = 1$) permits us to make the fullest use of this convergence, I have not yet met with any difficulty which could not be surmounted by putting $r = 3$.

In not a few of my applications I have certainly met with the peculiar drawback, which I then indicated, of the changes of sign in both numerator and denominator for nearly equal values of n within the observed intervals, but not as a detriment only. It has often warned me and made me give up certain

[1] In this article the formula at the top of p. 71 should read

$$\frac{(p+c)^2 - (m+c)^2}{\lambda_\infty - N} = \frac{(p+c)^2 - (n+c)^2}{P - N} - \frac{(n+c)^2 - (m+c)^2}{N - M}$$

hypotheses, whose incorrectness I could not have demonstrated without its aid.

But the formula is a cumbersome one. I do not know how many coördinate numbers may be necessary for an ideal chemistry to indicate all the properties of any particular substance, especially to compute the wave-length of every line of its spectrum, but that eight or even six constants of the formula for a series should be independent of one another, I do not believe. However, as long as I lack all knowledge of general relations suitable to reduce the law of series into a simpler form, I shall decidedly prefer a formula which satisfies the observations by means of many constants to one which, with only three or four constants, is unable to represent the observations approximately within the limits of their mean errors.

But unfortunately, not many spectra are rich enough in lines to permit the application of so cumbersome an apparatus of calculation, and at the same time not so complicated that it is impossible to separate their series. And of these only a few are sufficiently well known. My knowledge of the literature of this special subject being rather small, the only data I have found which are suitable for the calculation are the excellent measures of the carbon and cyanogen band-spectra by Messrs. Kayser and Runge.[1]

On these I have tried my formula, and computed the constants for a considerable number of series, of which I now publish only the part concerning the third band of the carbon spectrum. For the spectrum of cyanogen a photograph of Mr. Rydberg's was measured at the Copenhagen Observatory while I was engaged upon the preliminary computations, and these new measures promise to be valuable supplements to the measures of Kayser and Runge.

The carbon band ought not to wait for the cyanogen, though it requires further measurements before its calculation can be terminated definitively. The measures of Kayser and Runge have proved excellent in permitting a nearly complete resolution

[1] *Abhandlungen d. K. Akad. d. W., Berlin*, 1889, Anhang, p. 1.

of the band into its series. But as a little uncertainty remains in the computation of each series, it is possible that a revision of their old photographs (as to the existence of some lines probably overlooked) and the addition of the intensities of the lines might afford an essential improvement. I wish, however, to appeal to all observers by pointing out that here is a field where small special improvements in measuring give hopes of considerable results. A complete resolution of the heads themselves of this band, which is the radical remedy for the present uncertainty, may require an extraordinary dispersion, but every small increase in dispersion, if it only result in the resolution of a few of the coincident lines in Kayser and Runge's tables, especially near the heads of the band, may also possibly furnish the computer with the means of deciding among certain hypotheses now too equally probable. On the other hand, prolonged exposure of the photographs may reveal the existence of some important lines, or parts of series, in the faint half of the band (near the fourth band) which may have been too faint to appear on the photographs of Kayser and Runge.

The third band of the carbon spectrum contains not only the two heads indicated by Kayser and Runge, but certainly also two others, and probably many with rapidly decreasing intensity. The first head, $\lambda = 5165$, is very intense, the second, $\lambda = 5129$, rather faint, the third, $\lambda = 5100$?, the fourth, $\lambda = 5067$, and that which is probably the fifth, $\lambda = 5050$, show on Kayser and Runge's photograph no head-like appearance; most likely only the series from the first three heads are sufficiently intense to render visible their isolated lines, while in the others we see only those which are strengthened by coincidences.

The parts of the band belonging to each of these heads seem to consist of five or probably six complete series, or rather of two groups, each of three (or two) series, arranged in some analogous manner. The series of a group proceed with a pronounced tendency to parallelism, and in hardly less degree the groups themselves, also. Far from the heads this parallelism is most pronounced, and but few coincidences occur; but those

that occur continue for a considerable distance. Near the heads the coincidences set in, and soon become very complicated on account of the large number of series whose heads tend to coincide in the great common head of the band. These coincidences between the heads of the series cannot be determined by means of Kayser and Runge's photographs. A line, $\lambda = 5166.25$, which falls near the first great head, may possibly represent the head of a series; otherwise the only trace of multiplicity in the heads found by Kayser and Runge is that the second line of the band is marked "strong." Nor have I been able to decide by my computations, whether the five or six heads of series very nearly coincide in the common head, or whether their wave-lengths differ from one another by an Ångström unit or even more. This doubt is the principal cause of my request for new observations.

It is common to all the series of the carbon band that, as their phases are all nearly equal to $\frac{1}{4}$, their branches are very well separated. Indeed the phase is in general not exactly $\frac{1}{4}$. The halting progress of the complete series, the regular succession of longer and shorter intervals, has already struck Kayser and Runge, and made them complain of the slight regularity of a series and of the variability of its intervals.

In one of the groups of the first common head there are only two series, α and β, but α is much stronger than β, and in the other series I believe it is irresolvably double from head to tail. The β series is also a very close companion to α. Near the head, where they are most widely separated, Kayser and Runge invariably unite the measures of α and β with a $\left\{\right.$ as a sign of imperfect resolution. The coincidences set in with that of $\beta-16$[1] and $\alpha-16$, and apparently continue with rapid decrease of the intensity of the joint $\alpha\beta$ series until the lines of the positive branch disappear at $\alpha\beta+43$, and the last of the negative at $\alpha\beta-47$.

The other group of the first head, consisting of the series γ,

[1] β-16 is understood to mean the sixteenth line of the negative branch of the β series.

δ and ϵ, is a no less conspicuous feature, mentioned by Kayser and Runge, and described by them as a series of triplets occurring in the intervals of $\alpha\beta$. The γ series has constantly the shortest wave-length, δ the middle, and ϵ, with a somewhat greater distance, the longest wave-length. The middle series δ is first separated from the strong α and β headlines at $\delta+7$, then ϵ, while γ has no isolated line before $\gamma+11$. There are no later coincidences with $\alpha\beta$, but soon γ begins to coincide with δ (from $\gamma+17$ and $\delta+17$), and at last the coincident $\gamma\delta$ series also coincides with ϵ (from $\gamma\delta+29$ and $\epsilon+29$). By their junction these series acquire a great intensity, greater and much more permanent than that of $\alpha\beta$; they are seen by Kayser and Runge to the end of the band.

The second problem in this spectrum is to determine whether these coincidences remain irresolvable up to the end of the band. From Kayser and Runge we learn only that after apparent coincidence the lines cannot approach very rapidly, for in one instance, $\gamma\delta\epsilon-35$, Kayser and Runge have actually resolved the triplet, and seen the (ϵ ?) line isolated ; but with this exception it must be admitted that their measures show no trace of a division or of the separation of a diverging lateral branch. I have serious reason to believe, however, that such a separation takes place. Where the three series are isolated, and also after the coincidence of γ and δ, their phases are very nearly the same number, 0.244 (decidedly different from 0.250). But we observe with some surprise that, shortly after the total coincidence, the values of the phase begin to vary gradually, so that at last it attains the value $c=0.235$, which is beyond doubt really different from 0.244.

That the phase must remain constant in each series is my fundamental hypothesis. This, in fact, is not contradicted by this variability in the phase of something that manifestly is not a series but a combination of series. Consequently I have sought to make use of this phenomenon as an indication of the distribution of the single series in the compound. It is obvious that a greater variation of the phase of the compound series

must lead to the supposition of a considerable difference between the intensities of the branches of some of the coincident series. For the sake of computation I have formulated the necessary supposition as the following special hypothesis: The lines seen at the end of the band only belong to the two extreme branches $\gamma-n$ and $\epsilon+n$ of the six branches, so that they are here isolated again; both branches of the δ series have, I think, become invisible to Kayser and Runge about $\delta\pm50$; even before the positive branch of γ and the negative branch of ϵ have been lost sight of, say at $\gamma+40$ and $\epsilon-40$. Evidently this hypothesis requires confirmation by further observations. A prolonged exposure of the photographs may settle this question. The hypothetical lateral branches must be some 0.2 or 0.3 Ängström unit distant from the known main branches, consequently no greater dispersion is wanted, but only a greater intensity.

Among the feebler series of the second head we again find the same duplicities or triplicities, but only so much wider that they do not appear by a simple inspection of the photographs. To one group belong at least two series, ζ and ξ, of which (because of the great difficulties arising principally from the coincidences with the strong series from the first head) I have been able to compute only the ζ series, while the existence of the ξ series is evinced by too few isolated lines to permit an independent computation. I have found it only by a geometrical representation of the differences between the computed ζ lines and all those observed by Kayser and Runge. The ζ series itself may be double. Beyond $\zeta-37$ my ζ series loses its negative branch; the lines marked $\zeta-36$, $\zeta-39$, and the following ones in the catalogue of comparison below belong probably to a series ζ', intervening with ζ in a similar manner as γ with ϵ, whose existence is indicated also in the positive branch by some cases of apparent duplicity.

To the other group of the second head belong two computed series, η and θ, and a third series ι, traced out in the same manner as ξ by means of graphical comparison of η and θ with all the observed lines. These series, η and θ, coincide with $\eta\theta-32$,

and are lost sight of shortly after; traces of η? may perhaps be found as far as $\eta \pm 46$.

As to the third head, I know neither the exact value of its maximum wave-length nor the number of series issuing from it. By means of the graphical process, described below, it is easy to prove the existence of two such series, κ and λ. Of these λ can be computed, a fairly sufficient number of isolated (?) lines being observed by Kayser and Runge; but such a computation must be immature as yet, because a great number of additional lines are lacking in Kayser and Runge's tables.

Before giving the numerical results of my computation, however, I shall set forth some particulars of the operations. Indeed, the unraveling and computing of a series of a band spectrum differs in more than one regard from common practice in astronomical computations.

As soon as we have succeeded in discovering the first series of a band spectrum and in calculating the formula, $\lambda = f(n+c)^2$, it is possible to arrange the operations always necessary for its improvement in a way which gives hope that other series may be detected at the same time. With this formula a table of the function $f(m^2)$ is to be computed, say for the whole numbers of m; by interpolation in such a table the values of $f(n+c)^2$ can be found and compared with the observed values of λ, corresponding to the supposed series, all in the ordinary way. Nor is it too much to ask that in a somewhat difficult matter of this kind the computer shall not satisfy himself by the immediate inspection of the catalogue of differences Obs.—Comp., but also illustrate them by means of the ordinary construction, putting m as the abscissa and $\lambda_{\text{obs}} - f\,(m+c)^2$ as the ordinate of a point for each observation.

In our case the labor of interpolation with respect to the phase can be spared; on the other hand it may be very useful to extend the comparison of the observed values, without regard to their relation to the supposed series, to all observed lines, or practically to those differing from $f(m^2)$ by less than a convenient arbitrary quantity (*e.g.*, the maximum of $f(m^2) - f(m+1)^2$).

For every abscissa m, all the differences $\lambda_{obs}-f(m^2)$ are to be laid down as ordinates in the construction of points of observation. It may also be useful to lay down on each ordinate the points corresponding to $f(m+r)^2-f(m)^2$ within the arbitrary limit. A curve joining the points for the same r then represents both the abscissa and the wave-lengths of a series with the phase $=0$, but in all other respects similar to the supposed series.

The total figure formed by these curves has, for all band-spectra, the shape of an onion, cut through longitudinally. The curves all pass very nearly and with inflexion through the zero point of the system of coördinates or the head of the series; at some distance from this point they attain their maximum distance, and then approach again. The space between any two consecutive curves, the section of a single layer of the onion, contains one and only one point of observation for each observed line of the part of the spectrum in which the supposed series is found.

Being a two-dimensional representation of the spectrum, each of these spaces must show any of its series as a curve passing through the observed points of its ordinates; in particular, the supposed series employed in the construction ought to represent its two branches as two veins of the layer, running through its whole length symmetrically to its limits, so that on the two layers next to the abscissa its geometrical representation becomes very well fitted for graphical adjustment, with these special advantages that the two branches can be treated separately, that the value of the phase can be found repeatedly by the proportions of sections of each ordinate, that its constancy can be tried, and the phase finally can be eliminated in the formation of normal values for the subsequent computation.

The other series of the same spectrum are represented in a somewhat similar manner; only, in general, they may traverse the limiting lines of our onion figure and be more or less difficult to perceive. But as the adjacent series, following one another in a compound band, always seem to obey some continuous law, it may be expected that they only differ by small quantities,

and that their representations consequently do not differ greatly from those of the supposed series.

So in the carbon band; the series ϵ, which passes through the whole band, may be employed for comparison. Any of our formulæ for this series may be used. We may with 6^{mm} as unit for m on the abscissa and 15^{mm} representing an Ångström unit on the ordinates, obtain a fairly good representation, not only of the two branches of the ϵ series itself, but also of the δ and γ series which are represented by nearly parallel curves, only starting from somewhat distant heads. Moreover, the α and β series are still well represented, showing only a small inclination towards the three just mentioned. The series of the second head of the band experience greater distortion; near their heads, particularly, they cross the other series and the abscissa with nearly perpendicular transverse curves. Somewhat farther off, where these series no longer coincide, the inclinations of the curves become gradually smaller, and finally some of their curves may also be detected immediately by inspection.

Choosing one of these discovered series, say ζ, as the fundamental of a new geometrical representation, especially for the examination of the series from the second head, and repeating these operations as far as possible, we can resolve the whole band into its series, in a methodical manner, fairly applicable to most of the band-spectra. In such a way the series ξ, i, κ, and λ are actually discovered.

Moreover, this method supplies another advantage. This kind of induction is a somewhat hazardous one, something like the deciphering of a cryptograph. Only the final complete concordance affords security; in all the preparatory stages great mistakes threaten us, the worst mistake being to pass from one series into another coincident series, particularly if the coincidence is prolonged and more like a tangency than a simple intersection. For the criticism necessary to avoid such mistakes, we can scarcely be provided with any better instrument than our graphical representation illustrating on a single sheet all the series in question.

The constants of the computed series will not be published in a form corresponding directly to equation (1). For the computation I have with some little advantage put

$$\frac{1}{\lambda-\mu}=\phi=\frac{v_0+v_1(n+c)^2+\quad.\quad.\quad.\quad.\quad v_r(n+c)^{2r}}{p_0+p_1(n+c)^2+\quad.\quad.\quad.\quad.\quad 1\,.\,(n+c)^{2r}}, \tag{2}$$

μ being an estimated value of the wave-length λ_0 of the head, but this form is also unsuitable for publication. But knowing only very imperfectly the characteristic details of this application of our formula, I have greatly hesitated in my choice and ultimately preferred the form

$$\lambda=\lambda_0-\kappa\left(\frac{n+c}{10}\right)^2.\frac{1+s_1\left(\frac{n+c}{10}\right)^2+\quad.\quad.\quad+s_{r-1}\left(\frac{n+c}{10}\right)^{2r-2}}{1+t_1\left(\frac{n+c}{10}\right)^2+\quad.\quad.\quad+t_r\left(\frac{n+c}{10}\right)^{2r}}, \tag{3}$$

because it brings into relief the natural constants λ_0 and κ, showing the frame of the head, and moreover because it fairly illustrates the degree of convergence obtained. It may be remarked, however, that all the special values for s_i show a tendency to equality with t_i; taking this peculiarity into consideration, a further development,

$$\lambda=\lambda_0-\frac{\kappa}{\left(\frac{10}{n+c}\right)^2+\psi},$$

is indicated, where ψ is a function of the same class as λ, only with $r-1$ instead of r; if ψ be constant, the formula becomes identical with that of Professor Pickering, being thus the first step of a development in continuous fraction.

The results of my computations are as follows :

Series α		Series β	
$c=$	0.266	$c=$	0.266
$\lambda_0=$	5165.1733	$\lambda_0=$	5165.5911
$\kappa=$	11.53776	$\kappa=$	13.17201
$t_1=$	+4.523456	$t_1=$	+4.437308
$s_1=$	+5.049774	$s_1=$	+4.342970
$t_2=$	+0.1273115	$t_2=$	+0.1248869
$s_2=$	+0.1278293	$s_2=$	+0.1098562
$t_3=$	+0.000615434	$t_3=$	+0.000603713

The α and β constants are not found independently. The phase c has been determined from the coincident part of the series. Only two normal values for each series have represented isolated lines, and five common normal values for the coincident part were regarded as belonging to means $\frac{1}{3}(2\alpha + \beta)$ with double weights for the α series. Thus only two constants, v_0 and p_0 (with $\mu = 5165$), are determined separately for α and β, the other constants of formula (2) being common to both series. There are reasons to suppose that this choice, made in order to save labor, has not been fortunate as to the best representation of observations.

Series γ, I	Series γ, II	Series δ
$c =$ 0.245	$c =$ 0.255	$c =$ 0.2445
$\lambda_0 =$ 5163.7023	$\lambda_0 =$ 5164.4046	$\lambda_0 =$ 5164.9634
$\kappa =$ 13.73249	$\kappa =$ 13.03697	$\kappa =$ 15.28366
$t_1 =$ +0.150106	$t_1 =$ +0.005907	$t_1 =$ +0.668854
$s_1 =$ +0.140146	$s_1 =$ +0.000001	$s_1 =$ +0.579184
$t_2 =$ −0.0017029	$t_2 =$ +0.0000241	$t_2 =$ +0.0028975
$s_2 =$ −0.0022202		
$t_3 =$ −0.000008955		

Series ϵ, I	Series ϵ, II
$c =$ 0.2435	$c =$ 0.2565
$\lambda_0 =$ 5166.2211	$\lambda_0 =$ 5164.9003
$\kappa =$ 17.85338	$\kappa =$ 16.75553
$t_1 =$ +1.161822	$t_1 =$ +0.584760
$s_1 =$ +0.861099	$s_1 =$ +0.465199
$t_2 =$ −0.0186694	$t_2 =$ −0.0101763
$s_2 =$ −0.0178138	$s_2 =$ −0.0104149
$t_3 =$ −0.000104691	$t_3 =$ −0.000059560

Coincidences with the α and β series have forced me to take the first normal values for γ, δ, and ϵ so far from the heads that the number of lines between the heads and the nearest normal value cannot be fixed with accuracy. Both for γ and ϵ two hypotheses (I and II) concerning this number have been tried (by addition of a line in the case of γ, II, and by removing a line in that of ϵ, II). The most important results of these

alterations are found in the variations of λ_0 (0.7023 for γ and 1.3208 for ϵ) but these heads being invisible on Kayser and Runge's photographs, the question cannot be decided at present by that criterion. For the rest the two formulæ for ϵ do not show any real difference, both representing very well the totality of observed lines, and if the second formula for γ is certainly inferior to the first, the reason is only that it has been computed with $r=2$ (without s_2 and t_3), because by putting $r=3$ I met with one of those pleasant changes of sign in both the denominator and numerator of the formula. Until this question is settled, however, by new observations I am inclined to accord the leading place to γ, I, and ϵ, I, for the following secondary reasons, or perhaps presumptions: If the phases are for γ, $c=0.245$, for δ, $c=0.2445$, and for ϵ, $c=0.2435$, the three series may have really identical phases; not so with 0.255, 0.2445, and 0.2565. To this I may add that only in the combination γ, I, δ, and ϵ, I, each of the lines of the series corresponds naturally to one, and only to one, line of each other series, and in such a way in the long coincidences of these series that the coincident lines are also corresponding lines.

Series ζ		Series η		Series θ	
$c =$	0.241	$c =$	0.250	$c =$	0.233
$\lambda_0 =$	5130.5489	$\lambda_0 =$	5128.6955	$\lambda_0 =$	5129.4943
$\kappa =$	12.59299	$\kappa =$	8.83519	$\kappa =$	9.28698
$t_1 =$	-0.047702	$t_1 =$	$+1.516465$	$t_1 =$	$+1.288308$
$s_1 =$	-0.053423	$s_1 =$	$+2.026739$	$s_1 =$	$+1.655164$
$t_2 =$	$+0.0029518$	$t_2 =$	$+0.0046593$	$t_2 =$	$+0.0045931$
$s_2 =$	$+0.0032713$				
$t_3 =$	$+0.000017036$				

To these series, belonging to the second head of the band, the same remarks apply, only to a greater extent than for γ, δ, and ϵ. Two or more lines should perhaps be added or taken away near their heads.

Upon the whole it must be evident that these systems of constants are by no means to be considered as definitive and reliable for speculations regarding the true properties of the law

of spectral series. The main interest of my computations is not to be found in these constants, but in the tables of computed wave-lengths founded upon them. While my work in other points may call for many revisions, these tables will be useful for comparisons with the observations, present or future, in order to demonstrate the existence of all the more remarkable series of this spectrum.

Of these tables, the two tables for the more problematic formulæ γ, II and ϵ, II are here given directly, but only for the whole value of $n+c$:

n+c	λ (γ, II)	λ (ε, II)	n+c	λ (γ, II)	λ (ε, II)
0	5164.404	5164.900	32	5035.791	5028.049
1	64.274	64.733	33	27.958	20.022
2	63.883	64.233	34	19.925	11.801
3	63.231	63.408	35	11.697	03.388
4	62.319	62.266	36	03.277	4994.789
5	61.148	60.821	37	4994.668	86.006
6	59.717	59.083	38	85.875	77.041
7	58.027	57.065	39	76.902	67.904
8	56.080	54.776	40	67.754	58.597
9	53.875	52.225	41	58.434	49.121
10	51.414	49.419	42	48.947	39.483
11	48.698	46.362	43	39.298	29.686
12	45.727	43.058	44	29.491	19.735
13	42.504	39.510	45	19.530	09.634
14	39.030	35.720	46	09.421	4899.389
15	35.306	31.690	47	4899.168	89.003
16	31.334	27.421	48	88.776	78.481
17	27.115	22.914	49	78.251	67.828
18	22.652	18.171	50	67.597	57.048
19	17.946	13.193	51	56.819	46.147
20	13.000	07.982	52	45.923	35.131
21	07.815	02.539	53	34.913	24.004
22	02.395	5096.867	54	23.795	12.772
23	5096.740	90.967	55	12.574	01.442
24	90.855	84.842	56	01.256	4790.020
25	84.741	78.494	57	4789.845	78.515
26	78.401	71.925	58	78.347	66.935
27	71.838	65.139	59	66.767	55.292
28	65.056	58.138	60	55.112	43.601
29	58.057	50.925	61	43.386	31.882
30	50.844	43.504	62	31.595	20.166
31	43.421	35.878	63	19.744	08.504

The other eight are employed in the formation of the following catalogue of differences, "Observation — Computation," and may

easily be reconstructed by its means with three correct decimal places of the Ångström unit, and in their interpolated form.

In this catalogue, the first two columns contain the observations of Kayser and Runge, their remarks being given with the abbreviations used by themselves; *s* = strong, *d* = double, *u* = indistinct, *dgr* = dark ground, and $\{$ for imperfect separation. The catalogue has been completed with the hypothetical lines required by my formulæ, which are wanting, however, in Kayser and Runge's memoir; these insertions are marked with *hp* = hypothetical, and are conspicuous also by having three decimal places, while Kayser and Runge give only two. In the third column are given the designations of the computed line or lines as to their series, and for each of these lines (after *a*), the difference Obs.—Comp. The line

$$s, d\ 5133.79 \mid \alpha - 16,\ +.032;\ \beta - 16,\ -.048$$

for instance, indicates that the strong double line measured by Kayser and Runge at wave-length 5133.79 is a compound of the sixteenth lines of the negative branches of series α and series β; that α differs towards the violet side by 0.032, and β towards the red side by 0.048. Both the phases being $c = 0.266$, the tabular values are for $\alpha : n+c = -15.734$, $\lambda = 5133.758$, and for β: $n+c = -15.734$, $\lambda = 5133.838$. A fourth column is reserved for special remarks.

A statistical enumeration shows a total number of 524 lines in this catalogue, of which 479 are measured by Kayser and Runge, while 45 *hp* lines are hypothetical. Probably some of these are owing to errors in my interpretations of the behavior of the series, especially in the neighborhood of the heads; but having found more or less pronounced traces of most of these lines on the enlarged photograph, which accompanies the measures of Kayser and Runge, I must regard my hypotheses as previously corroborated, and attribute these discordances principally to accidental omissions by Kayser and Runge, which give hope that new observations, even though not necessarily more refined ones, will add a considerable number of new lines for the basis

of future computations. Indeed, ten years ago Kayser and Runge could not feel as great a call as we do now to extend the measures to the faintest lines, and to resolve as many duplicities as possible.

Nevertheless, some few lines are really missing in the observations, or only visible with very little intensity, in comparison to their neighbors in the series. Thus, for instance, the line 5048.978, whose absence is expressly mentioned by Kayser and Runge; on the Atlas photograph only a very faint line is visible at the wave-length where a strong triple line, $\gamma\delta\epsilon - 30$, is expected. Similar absences are found at 4922.236 for $\gamma\delta\epsilon + 44$ and 4891.583 for $\delta\epsilon + 47$, and $\zeta - 46$. In other spectra also similar omissions are remarked, and it has struck me that such an omission is relatively frequent for coincident lines, while I have never met with it with isolated lines. Without a thorough knowledge of the very fundamental law of the genesis of spectra, this phenomenon cannot be explained; but I am inclined to believe that, in analogy with the behavior of interfering rays of light, while coincident lines as a rule strengthen the common intensity, a very close coincidence may eventually result in a mutual counteracting and loss of intensity,

Among the lines measured by Kayser and Runge, 79 do not correspond to any of my eight computed series. So far the resolution of the spectrum into its series is not complete. But taking into account the results of my above-mentioned graphical construction by means of the table for the ζ series, I must conclude that the measured spectrum can contain but very few foreign lines. Upon the strength of this construction, 65 lines must be attributed to certain branches of certain series. A critical revision, based upon computations of these ξ, ι, κ, and λ series may probably exclude some of these lines; if so, they may, like the other 14 lines marked *N3* or μ? or ν?, most likely be attributed to series from the third head other than κ and λ, or to the feeble series from the fourth and fifth head, or like the line 5155.25, represent the survivors of the second carbon band within the domain of the third band.

CATALOGUE OF COMPARISON: OBSERVATION–COMPUTATION.

Observed by Kayser and Runge		Indication of series, and difference of wave-length	Special remarks
hp	5166.210	$\epsilon+0$	
hp	66.118	$\epsilon-1$	
hp	65.946	$\epsilon+1$	
hp	65.675	$\epsilon-2$	
I. head	65.30	$\beta+0,-.281$; $\beta-1,-.219$; $\beta+1-.079$; $\epsilon+2,-.036$	
s	65.12	$\alpha+0,-.047$; $\alpha-1,+.004$; $\alpha+1,+.127$; $\beta-2,-.075$	
	64.84	$\alpha-2,+.014$; $\beta+2,-.077$; $\delta+0,-114$; $\delta-1,-.035$; $\delta+1,+.114$; $\epsilon-3,-.054$	
	64.59	$\alpha+2,+.019$	
	64.46	$\beta-3,-.152$; $\delta-2,-.033$; $\epsilon+3,+.065$	
	64.28	$\alpha-3,-.006$; $\delta+2,+.083$	
	64.04	$\beta+3,-.156$	The first lines of β evidently require a negative correction
	63.87	$\alpha+3,-.025$; $\delta-3,+.059$; $\epsilon-4,+.076$	
	63.62	$\beta-4,-.150$; $\gamma+0,-.074$; $\gamma-1,-.004$	
	63.49	$\alpha-4,+.000$; $\gamma+1,+.000$; $\delta+3,+.121$	
d	63.16	$\beta+4,-.058$; $\gamma-2,-.120$; $\epsilon+4,+.010$	
	62.96	$\alpha+4,+.000$; $\gamma+2,-.051$; $\delta-4,+.127$	
	62.60	$\beta-5,-.072$; $\gamma-3,-.061$	
	62.41	$\alpha-5,-.023$; $\gamma+3,+.152$; $\delta+4,+160$; $\epsilon-5,+.010$	Perhaps $\gamma+3$ and $\delta+4$ ought to be referred to an hp line 62.254
u	61.95	$\beta+5,-.033$	
	61.77	$\alpha+5,+.005$; $\gamma-4,+.000$; $\epsilon+5,+.150$	
u	61.40	$\beta-6,+.082$; $\delta-5,-.168$?	
	61.23	$\gamma+4,-.003$	
	61.08	$\alpha-6,-.036$	
	60.79	$\delta+5,-.058$; $\epsilon-6,+.058$	
	60.43	$\beta+6,-.063$; $\gamma-5,-.174$	
	60.31	$\alpha+6,+.002$	
	59.92	$\gamma+5,-.015$; $\delta-6,-.105$; $\epsilon+6,+.096$	
	59.66	$\beta-7,-.048$	
	59.50	$\alpha-7,-.036$	
u	59.10	$\gamma-6,-.068$; $\delta+6,-.071$	
	58.69	$\beta+7,-.058$; $\epsilon-7,-.113$	
	58.58	$\alpha+7,-.008$	
u	58.17	$\gamma+6,-.197$; $\delta-7,-.040$	
	57.79	$\beta-8,-.054$; $\epsilon+7,+.016$	
	57.65	$\alpha-8,-.043$; $\gamma-7,+.186$	
u	57.24	$\delta+7,+.014$	
	56.71	$\beta+8,-.039$	
	56.61	$\alpha+8,+.002$; $\gamma+7,+.080$; $\epsilon-8,-.017$	
u	56.17	$\delta-8,+.039$	
	55.70	$\beta-9,-.026$	
	55.56	$\alpha-9,-.032$; $\gamma-8,+.070$; $\epsilon+8,+.082$	
u	55.25	N_3	From the second band?
u	55.07	$\delta+8,+.051$	
	54.49	$\beta+9,-.005$; $\gamma+8,+.064$	
	54.35	$\alpha+9,-.018$; $\epsilon-9,+143$	Only from about here, where most of the lines become isolated, a closer agreement with the observations can fairly be expected
u	53.82	$\delta-9,+.028$	
	53.32	$\beta-10,-.033$	
	53.21	$\alpha-10,-.022$; $\gamma-9,-.041$	

CATALOGUE OF COMPARISON: OBSERVATION — COMPUTATION—*cont'd.*

Observed by Kayser and Runge		Indication of series, and difference of wave-length	Special remarks
	5152.97	ε + 9, +.027	
u	52.56	δ + 9, +.004	
	51.97	β + 10, —.018; γ + 9, —.088	
	51.87	α + 10, —.003	
	51.57	ε — 10, +.022	
	51.22	δ — 10, +.021	
	50.73	β — 11, +.002; γ — 10, —.018	
	50.61	α — 11, —.007	
	50.20	ε + 10, +.032	
	49.83	δ + 10, —.009	
u	49.33	β + 11, +.102; γ + 10, —.096	
s	49.14	α + 11, +.018	
	48.65	ε — 11, —.002	
	48.36	δ — 11, +.006	
u	47.89	β — 12, +.040; γ — 11, —.093	
s	47.73	α — 12, —.018	
	47.15	ε + 11, —.005	
	46.85	δ + 11, —.022	
	46.52	γ + 11, —.013	
	46.23	β + 12, +.014	
s	46.13	α + 12, +.012	
	45.52	ε — 12, +.002	
	45.27	δ — 12, +.010	
	44.98	γ — 12, +.020	
	44.72	β — 13, —.001	
s	44.62	α — 13, —.006	
	43.91	ε + 12, +.005	
	43.64	δ + 12, —.016	
	43.37	γ + 12, —.012	
	42.98	β + 13, +.026	
s	42.89	α + 13, +.028	
	42.15	ε — 13, +.004	
	41.93	δ — 13, +.012	
	41.69	γ — 13, +.013	
	41.37	β — 14, +.028	
s	41.26	α — 14, +.008	
	40.41	ε + 13, —.007	
	40.18	δ + 13, —.016	
	39.95	γ + 13, —.026	
	39.49	β + 14, +.048	
s	39.39	α + 14, +.034	
	38.56	ε — 14, +.025	
	38.34	δ — 14, +.007	
	38.13	γ — 14, —.010	
	37.72	β — 15, +.006	
s	37.62	α — 15, —.010	
	36.71	ε + 14, +.020	
	36.47	δ + 14, —.020	
	36.31	γ + 14, —.005	
	35.70	β + 15, +.017	
s	35.63	α + 15, +.029	
	34.70	ε — 15, +.014	

CATALOGUE OF COMPARISON: OBSERVATION — COMPUTATION—*cont'd.*

Observed by Kayser and Runge		Indication of series, and difference of wave-length	Special remarks
	5134.53	$\delta-15,+.028$	
	34.34	$\gamma-15,-.010$	
s, d	33.79	$\alpha-16,+.032$; $\beta-16,-.048$	
	32.74	$\epsilon+15,+.014$	
	32.52	$\delta+15,-.021$	
	32.40	$\gamma+15,-.002$	
s, d	31.68	$\alpha+16,+.082$; $\beta+16,+.004$	
	30.62	$\epsilon-16,+.022$	
	30.46	$\delta-16,+.030$; $\zeta+0,-.082$; $\zeta-1,-.017$	The Atlas photograph of K. & R. agrees well with the assumption of a head of a single series at 30.5, the interval between $\delta-16$ and $\gamma-16$ being darker, and particularly one small line being visible in the interval between $\gamma-16$ and $\alpha-17$, otherwise very clear
	30.32	$\gamma-16,+.010$; $\zeta+1,-.035$; $\zeta-2,+.159$	
hp	29.918	$\zeta+2$	
s	29.67	$\alpha-17,+.029$; $\beta-17,-.047$; $\zeta-3,+.078$	
II. head	29.36	$\theta+0,-.129$; $\theta-1,-.080$; $\theta+1+.008$	
	29.20	$\zeta+3,-.028$; $\theta-2,-.001$	
	28.93	$\theta+2,-.093$	
	28.72	$\zeta-4,-.052$; $\eta+0,+.028$; $\eta-1,+.074$; $\theta-3,-.045$	The head of the η series at 287 agrees with the d. gr of K. & R. But an interchange of the heads of ζ, η, and θ is possible
d. gr.	28.51	$\epsilon+16,-.012$; $\eta+1,-.046$; $\eta-2,+.089$; $\theta+3,+.019$	
	28.23	$\gamma+16,-.010$; $\delta+16,-.121$; $\zeta+4,-.057$; $\eta+2,-.008$	
	28.02	$\eta-3,+.016$; $\theta-4,-.099$	
	27.73	$\zeta-5,+.028$; $\eta+3,+.011$; $\theta+4,-.012$	
	27.38	$\alpha+17,+.029$; $\beta+17,-.045$; $\eta-4,+.001$	
	27.26	$\theta-5,+.012$	
	27.03	$\zeta+5,-.066$; $\eta+4,+.046$	
	26.88	$\xi+?$ $\iota-?$	The ι and ξ series are not computed
	26.73	$\theta+5,-.033$	
hp	26.531	$\eta-5$	
	26.30	$\epsilon-17,+.027$; $\zeta-6,-.081$	
s	26.13	$\delta-17,+.013$; $\theta-6,-.013$	
	26.04	$\gamma-17,+.017$; $\eta+5,+.020$	
	25.71	$\zeta+6,+.054$	
	25.53	$\eta-6,+.083$; $\theta+6,-.015$	
s	25.30	$\alpha-18,+.021$; $\beta-.18,-.052$	
	24.90	$\xi-?$ $\iota+?$	
	24.82	$\zeta-7,+.008$; $\eta+6,+.006$; $\theta-7,+.025$	
s	24.11	$\epsilon+17,+.029$; $\eta-7,-.009$; $\theta+7,+.027$	
s	23.87	$\gamma+17,+.038$; $\delta+17,-.050$; $\zeta+7,-.097$	
	23.34	$\eta+7,-.022$	
	23.21	$\theta-8,+.010$	
s	22.88	$\alpha+18,+.020$; $\beta+18,-.050$; $\zeta-8,-.114$	
	22.46	$\eta-8,-.083$	
	22.36	$\theta+8,-.012$	
s	21.76	$\epsilon-18,+.050$; $\zeta+8,-.270$; $\eta+8,+.098$	Perhaps rather hp 22.030 for $\zeta+8$
s, d	21.52	$\gamma-18,+.030$; $\delta-18,-.044$; $\theta-9,+.164$	
	20.71	$\alpha-19,+.036$; $\beta-19,-.034$; $\zeta-9,-.220$ $\eta-9,-.009$	Perhaps rather hp 20.930 for $\zeta-9$
	20.39	$\theta+9,-.022$	
u	19.72	$\zeta+9,-.127$; $\eta+9,+.007$	

CATALOGUE OF COMPARISON: OBSERVATION — COMPUTATION — *cont'd.*

Observed by Kayser and Runge		Indication of series, and difference of wave-length	Special remarks
	5119.40	ε+18,—.003	
s, d	19.21	γ+18,+.030; δ+18,—.041; θ—10,—.053	
	18.85	ζ—10,+.229; η—10,+.204	N 3
s	18.17	α+19,+.043; β+19,—.025; θ+10,—.033	
	18.08	ι—?	
	17.38	ζ+10,—.041; η+10,—.137	
	16.93	ε—19,+.018; θ—11,+.007	
s, d	16.74	λ—19,+.026; δ—19,—.034	
	16.30	η—11,—.027	
s	15.84	α—20,+.010; β—20,—.057; ζ—11,—.228; θ+11,+.092	Perhaps rather hp 16.068 for ζ—11
	14.99	η+11,—.085	
hp	14.751	ζ+11	This ζ line seems to be visible on K. & R.'s Atlas photograph; its observation and the resolution of some of the neighboring ζ coincidences are important
	14.48	ε+19,—.010	
s	14.31	γ+19,+.024; δ+19,—.035; θ—12,—.027	
	13.76	η—12,—.003	
s	13.17	α+20,+.015; β+20,—.050; ζ—12,—.102; θ+12,+.121	
	12.41	η+12,+.019	
	11.87	ε—20,—.008; ζ+12,+.031	
s	11.71	γ—20,+.012; δ—20,—.037	
	11.42	θ—13,—.088	
s	10.77	α—21,+.024; β—21,—.041; η—13;—.189	
u	10.10	ζ—13,—.137; θ+13,—.007	
	09.79	ι—?	
	09.35	ε+20,+.009; η+13,—.117	
s	09.17	γ+20,+.016; δ+20,—.033	
u	08.45	ζ+13,—.239; θ—14,+.012	Perhaps rather hp 08.689 for ζ+13
s	07.97	α+21,+.025; β+21,—.039; η—14,+.055	
	07.67	ξ—?	
d	06.98	ζ—14,+.016; θ+14,+.055	
	06.60	ε—21,—.009	
s	06.44	γ—21,—.004; δ—21,—.045; η+14,+.136	
	06.00	ξ+?	
	05.44	α—22,+.014; β—22,—.049; ζ+14,+.139	
u	05.11	θ—15,—.019	
	04.67	η—15,+.035	
	03.95	ε+21,—.010	
s	03.80	γ+21,+.016; δ+21,—.028	
	03.43	ζ—15,—.024; θ+15,—.079	
	03.17	ι—?	
	02.93	η+15,+.023	
s	02.53	α+22,+.029; β+22,—.032	
	01.58	ζ+15,—.098; θ—16,—.006	
	01.10	ε—22,—.010; η—16,—.021	
s	00.95	γ—22,—.006; δ—22,—.042	
s	5099.89	α—23,+.017; β—23,—.043; ζ—16,+.181; θ+16,+.036	
	99.27	η+16,—.006	
	98.34	ε+22,—.007	
s	98.19	γ+22,+.009; δ+22,—.032	
u	97.80	ζ+16,—.022; θ—17,—.008	The indistinctness of

CATALOGUE OF COMPARISON: OBSERVATION — COMPUTATION—*cont'd.*

Observed by Kayser and Runge		Indication of series, and difference of wave-length	Special remarks
u	5097.51	$\iota + ?$	this and the following lines may indicate the approximate position of the III head
u	97.36	$\eta - 17, -.014$	
s	96.84	$\alpha + 23, +.016;\ \beta + 23, -.043$	
u	95.98	$\theta + 17, +.011$	
hp	95.734	$\zeta - 17$	
	95.36	$\epsilon - 23, -.019;\ \eta + 17, -.055$	
s	95.22	$\gamma - 23, -.015;\ \delta - 23, -.049$	
	94.83	$\xi + ?$	
s	94.13	$\alpha - 24, +.042;\ \beta - 24, -.017$	
	93.74	$\zeta + 17, +.005;\ \theta - 18, -.060$	
u	93.45	$\eta - 18, +.051$	
u	92.88	$\xi - ?$	
	92.52	$\epsilon + 23, +.014$	
s	92.36	$\gamma + 23, +.012;\ \delta + 23, -.027$	
	91.85	$\theta + 18, -.005$	
	91.51	$\zeta - 18, -.018$	
u	91.29	$\eta + 18, -.035$	
s	90.94	$\alpha + 24, +.022;\ \beta + 24, -.035$	
u	90.51	$\xi + ?$	
	89.43	$\epsilon - 24, +.008;\ \zeta + 18, +.009;\ \theta - 19, -.134$	
s	89.29	$\gamma - 24, +.006;\ \delta - 24, -.028;\ \eta - 19, +.095$	
	88.55	$\xi - ?$	
	88.11	$\alpha - 25, +.034;\ \beta - 25, -.022$	
	87.53	$\theta + 19, +.017$	
	87.09	$\zeta - 19, -.007$	
	86.91	$\eta + 19, -.099$	
	86.43	$\epsilon + 24, -.009$	
s	86.31	$\gamma + 24, +.024;\ \delta + 24, -.015$	
	85.12	$\theta - 20, +.018$	
s	84.80	$\alpha + 25, +.015;\ \beta + 25, -.041;\ \zeta + 19, -.081;\ \eta - 20, +.033$	
u	83.93	$\xi - ?$	
	83.24	$\epsilon - 25, +.001$	
s	83.08	$\gamma - 25, -.026;\ \delta - 25, -.060;\ \theta - 20, +.134$	
u	82.35	$\zeta - 20, -.091;\ \eta + 20, -.119$	
s	81.86	$\alpha - 26, +.022;\ \beta - 26, -.033$	
	81.42	$\xi + ?$	
	80.45	$\theta - 21, +.034$	
	80.15	$\epsilon + 25, +.003;\ \zeta + 20, +.034;\ \eta - 21, +.035$	
s	80.03	$\gamma + 25, +.032;\ \delta + 25, -.009$	
s	78.44	$\alpha + 26, +.012;\ \beta + 26, -.043$	
	78.16	$\theta + 21, +.003$	
	77.70	$\eta + 21, -.006$	
	77.52	$\zeta - 21, -.043$	
	76.83	$\epsilon - 26, -.002$	
s	76.70	$\gamma - 26, -.006;\ \delta - 26, -.041$	
	75.42	$\alpha - 27, +.043;\ \beta - 27, -.011;\ \theta - 22, -.090$	
	75.03	$\zeta + 21, -.103;\ \eta - 22, -.212$	N 3
	73.64	$\epsilon + 26, +.006$	
	73.53	$\gamma + 26, +.042;\ \delta + 26, -.001$	
	73.16	$\theta + 22, +.012$	
	72.61	$\eta + 22, -.113$	

CATALOGUE OF COMPARISON: OBSERVATION — COMPUTATION—*cont'd.*

Observed by Kayser and Runge		Indication of series, and difference of wave-length	Special remarks
u	5072.48	$\zeta - 22, +.014$	
s	71.88	$\alpha + 27, +.030$; $\beta + 27, -.024$	
d	70.46	$\theta - 23, +.076$	
	70.20	$\epsilon - 27, -.007$; $\eta - 23, +.050$	
s	70.08	$\gamma - 27, -.004$; $\delta - 27, -.042$	
	69.86	$\zeta + 22, -.072$	
s	68.73	$\alpha - 28, +.034$; $\beta - 28, -.020$	
	68.28	$\lambda - ?$	The λ series belonging to the third head is not computed
	67.91	$\theta + 23, -.012$	
u	67.59	$\eta + 23, +.068$	
hp	67.151	$\zeta - 23$	
	66.91	$\epsilon + 27, +.008$	
s	66.81	$\gamma + 27, +.050$; $\delta + 27, +.005$	
	66.46	$\lambda + ?$ $\mu ?$	The μ lines belong probably to some uncomputed series from the IV head
	66.32	$\mu ?$	
s	66.07	$\xi + ?$ $\mu ?$	
	65.56	$\mu ?$	
	65.41	$\mu ?$	
	65.18	$\mu ?$	
s	65.09	$\alpha + 28, +.036$; $\beta + 28, -.016$; $\theta - 24, +.047$	
hp	64.841	$\eta - 24$	
u	64.59	$\zeta + 23, +.075$	
	64.32	$\iota + ?$ $\mu ?$	
	63.67	$\xi - ?$ $\mu ?$	
	63.39	$\epsilon - 28, +.024$	
hp	63.265	$\gamma - 28 (+.021)$; $\delta - 28, (-.020)$	Overlooked by K. & R.; very strong on their Atlas photograph
	62.46	$\theta + 24, -.020$	
hp	62.106	$\eta + 24$	
	61.81	$\alpha - 29, +.010$; $\beta - 29, -.042$	
	61.53	$\zeta - 24, -.096$	
	59.94	$\epsilon + 28, -.015$	
	59.85	$\gamma + 28, +.036$; $\delta - 28, -.013$	
u	59.34	$\eta - 25, +.023$; $\theta - 25, -.147$	
	58.91	$\zeta + 24, +.024$	
s	58.06	$\alpha + 29, +.016$; $\beta + 29, -.035$	
	56.80	$\theta + 25, -.026$	
	56.30	$\epsilon - 29, -.008$; $\eta + 25, -.176$	
s	56.21	$\gamma - 29, +.020$; $\delta - 29, -.024$	
u	55.88	$\zeta - 25, -.006$	
s	54.73	$\alpha - 30, +.040$; $\beta + 30, -.011$	
	54.37	$\lambda + ?$	
	53.66	$\eta - 26, +.079$; $\theta - 26, -.060$	
	53.14	$\zeta + 25, +.094$	
s	52.75	$\gamma + 29, +.094$; $\delta + 29, +.042$; $\epsilon + 29, -.045$	
s, u	50.86	$\alpha + 30, +.037$; $\beta + 30, -.012$; $\eta + 26, +.226$; $\theta + 26, -.101$	
	49.89	$\zeta - 26, -.048$	
s	49.68	$\lambda + ?$ $\nu ?$	The ν lines belong probably to some uncomputed series from the V head
	49.52	$\nu ?$	
hp	48.978	$\gamma - 30 (+.054)$; $\delta - 30 (+.008)$; $\epsilon - 30, (-.063)$	Remarkable absence of 3 coincident lines. Instead of the expected strong line, the Atlas photograph has only a faint trace of a line
	48.61	$\xi + ?$ $\nu ?$	
	48.46	$\nu ?$	
	48.27	$\nu ?$	

CATALOGUE OF COMPARISON: OBSERVATION—COMPUTATION—*cont'd.*

Observed by Kayser and Runge		Indication of series, and difference of wave-length	Special remarks
u	5047.68	$\eta - 27, +.046$; $\theta - 27, -.064$	
	47.41	$\alpha - 31, +.038$; $\beta - 31, -.011$	
	47.16	$\zeta' + ?$ $\lambda - ?$ $\nu ?$	The ζ' lines seem to indicate a duplicity of the ζ series
	47.02	$\zeta + 26, +.022$	
s	45.39	$\gamma + 30, +102$; $\delta + 30, +.047$; $\epsilon + 30, -.036$	
	44.87	$\theta + 27, -.019$	
hp	44.582	$\eta + 27$	
u	43.81	$\zeta - 27, +.024$	
u	43.42	$\alpha + 31, +.026$; $\beta + 31, -.023$	
s	41.47	$\gamma - 31, +.020$; $\delta - 31, -.030$; $\epsilon - 31, -.096$; $\eta - 28, -.009$; $\theta - 28, -.092$	
u	40.86	$\zeta + 27, +.113$	
	40.54	$\iota + ?$	
s	39.88	$\alpha - 32, +.032$; $\beta - 32, -.016$	
u	38.60	$\theta + 28, -.012$	
	38.22	$\eta + 28, -.104$	
s	37.82	$\gamma + 31, +.108$; $\delta + 31\ +.048$; $\epsilon + 31, -.032$	
	37.57	$\iota - ?$ $\lambda - ?$	
	37.42	$\zeta - 28, -.009$	
u	35.79	$\alpha + 32, +.027$; $\beta + 32, -.020$	
u	35.14	$\eta - 29, +.022$; $\theta - 29, -.037$	
	34.46	$\zeta' + ?$	
	34.27	$\zeta + 28, -.023$	
s	33.84	$\gamma - 32, +.069$; $\delta - 32, +.015$; $\epsilon - 32, -.048$	
d	33.68	$\kappa + ?$	The uncomputed κ series belongs to the third head; the duplicity of 33.68 is probably due to $\gamma - 32$
	33.08	$\xi - ?$	
s	32.18	$\alpha - 33, +.058$; $\beta - 33, +.011$; $\theta + 29, +.048$	
d	31.91	$\eta + 29, +.051$	
hp	30.871	$\zeta - 29$	
	30.48	$\kappa - ?$	
s	30.05	$\gamma + 32, +116$; $\delta + 32, +.051$; $\epsilon + 32, -.024$	
	29.60	$\lambda + ?$	
u	28.54	$\eta - 30, -.013$; $\theta - 30, -.050$	
	27.94	$\alpha + 33, +.008$; $\beta + 33, -.037$	
	27.65	$\zeta + 29, +.012$	
s, d	25.92	$\gamma - 33, +.030$; $\delta - 33, -.029$; $\epsilon - 33, -.087$	
u	25.49	$\theta + 30, +.037$	
hp	25.195	$\eta + 30$	
	24.22	$\alpha - 34, +.021$; $\beta - 34, -.024$	
s	24.09	$\zeta - 30, -.023$	
s	22.07	$\gamma + 33, +.114$; $\delta + 33, +.045$; $\epsilon + 33, -.026$	
	21.72	$\eta - 31, -.067$; $\theta - 31, -.085$	
	20.89	$\zeta' + ?$ $\lambda - ?$	
	20.79	$\zeta + 30, +.004$	
	19.87	$\alpha + 34, -.034$; $\beta + 34, -.079$	
u	18.58	$\theta + 31, +.003$	
hp	18.329	$\eta + 31$	
s, d	17.83	$\gamma - 34, +.018$; $\delta - 34, -.045$; $\epsilon - 34, -.100$	
	17.28	$\iota - ?$	
	17.13	$\zeta - 31, -.030$	
	16.12	$\alpha - 35, +.038$; $\beta - 35, -.005$	
u	15.29	$\lambda - ?$	

CATALOGUE OF COMPARISON: OBSERVATION — COMPUTATION—*cont'd.*

Observed by Kayser and Runge		Indication of series, and difference of wave-length	Special remarks
u	5014.84	$\eta-32,+.019;\ \theta-32,+.015$	
s	13.89	$\gamma+34,+.106;\ \delta+34,+.034;\ \epsilon-34,-.036;\ \zeta+31,+.151$	
u	12.42	$\xi-?\ \lambda+?$	
	11.66	$\alpha+35,-.026;\ \beta+35,-.068;\ \theta+32+.154$	
hp	11.265	$\eta+32$	
	10.03	$\zeta-32,+.017$	
	09.62	$\epsilon-35,-.038$	
s	09.53	$\gamma-35,-.010;\ \delta-35,-.076$	
	09.18	$\lambda-?$	
	07.82	$\alpha-36,+.046;\ \beta-36,+.003;\ \eta-33,+.160;\ \theta-33,+.168$	
u	07.27	$\kappa-?$	
	06.50	$\zeta+32,+.000$	
	06.24	$\iota+?\ \lambda+?$	
s, d	05.55	$\gamma+35,+.133;\ \delta+35,+.058;\ \epsilon+35,-.013$	
	04.37	$\theta+33,+.126$	
hp	04.006	$\eta+33$	
	03.20	$\alpha+36,-.078;\ \beta+36,-.121$	
d	02.68	$\zeta-33,+.005$	
s, d	01.09	$\gamma-36,+.014;\ \delta-36,-.055;\ \epsilon-36,-.110$	
	00.32	$\eta-34,+.016;\ \theta-34+.031$	
	4999.91	$\lambda+?$	
	99.65	N 3	For the lines marked N 3 no special indication is possible, but they may belong to the series from third head
	99.32	$\alpha-37,+,039;\ \beta-37,-.003$	
u	99.09	$\zeta+33,+.019$	
s, d	96.99	$\gamma+36,+.129;\ \delta+36,+.050;\ \epsilon+36,-.024;\ \theta+34,+.196$	
hp	96.554	$\eta+34$	
u	95.16	$\zeta-34,+.011$	
u	94.68	$\alpha+37,-.008;\ \beta+37,-.050$	
	93.39	$\lambda+?$	
	92.89	$\eta-35,+.133;\ \theta+35,+.151$	
s, d	92.44	$\gamma-37,+.013;\ \delta-37,-.058;\ \epsilon-37,-.113$	
	91.50	$\zeta+34,+.044$	
	91.12	$\iota+?$	
	90.64	$\alpha-38,+.034;\ \beta-38,-.008$	
u	90.12	$\xi-?\ \lambda-?$	
hp	89.035	$\eta+35\ (+.123);\ \theta+35,(-.123)$	The θ series is vanishing, the η series perhaps also, though the graphical construction seems to indicate a prolongation of both branches. The following η lines are given here only for use in a future investigation upon this question
s	88.27	$\gamma+37,+.150;\ \delta+37,+.068;\ \epsilon+37,-.004$	
	87.44	$\zeta-35,+.001$	
u	86.70	$\lambda+?$	
	85.96	$\alpha+38,+.040;\ \beta+38,+.000$	
hp	85.013	$\eta-36\ (-.008);\ \theta-36,(+.008)$	
s	83.62	$\gamma-38,+.026;\ \delta-38,-.046;\ \epsilon--38,-.102;\ \zeta+35,-.037$	
	81.79	$\alpha-39,+.034;\ \beta-39,-.006$	
hp	81.211	$\eta+36\ (+.128);\ \theta+36,(-.128)$	
s	79.36	$\gamma+38,+.160;\ \delta+38,+.077;\ \epsilon+38,+.002;\ \zeta-36,-.186$	The negative branch of ζ seems to vanish
	76.97	$\alpha+39,-.006;\ \beta+39,-.046;\ \eta-37,-.128;\ \theta-37,-.120$	

CATALOGUE OF COMPARISON: OBSERVATION — COMPUTATION—*cont'd.*

Observed by Kayser and Runge		Indication of series, and difference of wave-length	Special remarks
u	4975.69	ζ+36,+.012	
s	74.58	γ—39,—.002; δ—39,—.076; ε—39,—.135	
	73.69	η+38?	
	72.78	α—40,+.048; β—40,+.008	
	71.54	ζ—37,+.064	
s	70.25	γ+39,+.148; δ+39,+.064; ε+39,—.016	
hp	68.992	η—38	
	67.84	α+40,—.024; β+40, —.062	
	67.53	ζ+37,+.006	
s	65.39	γ—40,—.004; δ—40,—.080; ε—40,—.144	
hp	64.871	η+38	
	63.60	α—41,+.058; β—41,+.021	
hp	63.233	ζ—38	
	63.02	ι—?	
s	60.96	γ+40,+.129; δ+40,+.046; ε+40,—.040; η—39,+.255?	
	59.19	ζ+38,—.009	
	58.59	α+41,+.005; β+41,—.032	
	58.16	λ+?	
	57.73	ξ—?	
	57.42	η+39,+.926?	
s	56.08	γ—41,+.042; δ—41,—.031; ε—41,—.102	
	54.70	ζ—39,—.120?	ζ—39 and the following ζ lines marked ? do not really belong to the negative branch of ζ; probably the duplicity indicated by the ζ' lines of the positive branch will explain the anomalies of these lines
	54.25	α—42,+.063; β—42,+.026	
hp	52.239	η—40?	
s	51.50	γ+41,+.109; δ+41,+.026; ε+41,—.068	
	50.69	ζ+39,—.015	
	50.20	ι+?	
	49.14	α+42,—.006; β+42,—.042	
hp	47.940	η+40?	
s	46.46	γ—42,—.053; δ—42,—.124; ε—42,—.204	
	46.08	ζ—40,—.161?	
	44.69	α—43,+.016; β—43,—.021; η—41,+1.092	
	42.94	λ+?	
	42.62	N 3	
	41.92	γ+42,+.133; δ+42,+.054; ε+42;—.051; ζ+40,—.129	
	40.90	κ+?	
hp	39.212	η+41?	
	37.37	ζ—41,—.133?	
	36.83	γ—43,+.003; δ—43,—.064; ε—43,—.156	
	35.11	α—44,+.102; β—44,+.066	
hp	34.783	η—42?	
	33.27	ζ+41,+.035	
	32.18	γ+43,+.158; δ+43,+.084; ε+43,—.043	
hp	30.312	η+42?	
	28.52	ζ—42,—.089?	
	26.96	γ—44,—.022; δ—44,—.082; ε—44,—.189	
	25.79	η—43?	
hp	25.204	α—45 (+021); β—45,(—.011)	
	24.87	κ+?	

CATALOGUE OF COMPARISON: OBSERVATION — COMPUTATION—*cont'd.*

Observed by Kayser and Runge		Indication of series, and difference of wave-length	Special remarks
	4924.28	ζ + 42, +.011	
hp	22.236	γ + 44, +.134; δ + 44, (+.068); ε + 44, (—.068).	Absence of coincident lines
hp	21.244	η + 43 ?	
hp	19.566	ζ — 43 ?	
	18.05	η — 44, + 1.402	
	16.96	γ — 45, —.024; δ — 45, —.074; ε — 45, —.200	
	15.16	α —46, —.074; β—46, —.108; ζ+43, +.004	
	14.63	λ - - ?	The α and β series are totally lost
	12.23	γ + 45, +.199; δ+45, +.144; ε + 45, —.012; η+44, +.220 ?	
hp	10.379	ζ — 44 ?	
hp	07.332	η — 45 ?	
	06.86	γ — 46, +.023; δ—46, —.014	A considerable number of the last lines belonging to the positive branch of γ and to the negative branch of ε, though certainly invisible, have hitherto been taken in consideration of future observations; now the δ series also fades away
	05.88	ζ + 44, —.021	
	05.42	N3	
hp	02.614	η + 45 ?	
	01.96	δ + 46, +.108; ε+46—.072	
	00.90	ζ — 45, —.153 ?	
	4899.98	η — 46, +2.125 ?	
	97.56	λ — ?	
	96.52	γ—47, —.026; δ—47, —.046; ζ+45, +.010	
	93.72	λ + ?	
hp	93.057	η + 46 ?	
hp	91.583	δ + 47, (+.109); ε+47, (—.098); ζ—46; (—.010)	Absence of coincident lines
	90.89	η — 47, +2.671	See remark to 4989.035
	87.01	ζ + 46, +.023	
s	86.14	γ —48, +.023; δ—48, +.026	
	85.64	N	
	85.05	λ + ?	
hp	82.005	ζ — 47 ?	
	81.19	ε + 48, —.003	
	77.33	ζ+ 47, —.010	
	75.51	γ — 49, - .042; δ — 49, —.012	
hp	72.295	ζ — 48 ?	
	70.58	ε + 49, +.008	
	67.52	ζ + 48, —.052	
	64.86	γ — 50, +.000; δ — 50, +.066	
hp	62.467	ζ — 49 ?	
	59.88	ε + 50, +.056	
	58.55	N3	
	57.68	ζ + 49, —.009	
	55.95	κ + ?	
s	54.11	γ — 51, +.067	
	53.67	λ — ?	
	52.44	ζ — 50, —.085 ?	
s	48.93	ε + 51, —.023	
	47.66	ζ + 50, —.035	
	43.11	γ — 52, +.001	

CATALOGUE OF COMPARISON: OBSERVATION — COMPUTATION—*cont'd.*

Observed by Kayser and Runge		Indication of series, and difference of wave-length	Special remarks
	4842.31	$\zeta - 51, -.168$?	
s	37.99	$\epsilon + 52, +.025$	
	37.59	$\zeta + 51, -.008$	
	32.80	$\kappa -$?	
	32.13	$\gamma - 53, -.068$; $\zeta - 52, -.197$?	
	27.38	$\zeta + 52, -.020$	
	26.87	$\epsilon + 53, +.004$	
	25.88	$\lambda -$?	
	21.80	$\zeta - 53, -.279$?	
	20.93	$\gamma - 54, +.022$	
	17.14	$\zeta + 53, +.035$	
	15.66	$\epsilon + 54, +.000$	
	11.99	N3	
	11.50	$\zeta - 54, -.237$?	
s	09.63	$\gamma - 55, -.026$	
hp	06.720	$\zeta + 54$	
s	04.35	$\epsilon + 55, -.004$	
	01.03	$\zeta - 55, -.277$?	
	4798.79	$\kappa +$?	
s	98.32	$\gamma - 56, +.011$	
	96.24	$\zeta + 55, -.009$	
	92.92	$\epsilon + 56, -.036$	
	86.88	$\gamma - 57, +.004$	
	85.63	$\zeta + 56, -.064$	The last trace of the ζ series
	81.46	$\epsilon + 57, -.013$	
	79.44	$\kappa +$?	
	75.32	$\gamma - 58, -.044$	
	72,18	$\lambda +$?	
	69.87	$\epsilon + 58, -.044$	
	63.86	$\gamma - 59, +.078$	
	58.33	$\epsilon + 59, +.041$	
	52.06	$\gamma - 60, -.082$	
	46.55	$\epsilon + 60, -.063$	Some more lines of tne fourth band seem to belong to the γ and ϵ series

Of the 433 computed lines, no less than 229 are in the catalogue represented as isolated lines. In consideration of the great number of series, and the complicated structure of the spectrum, this proportion must be regarded as very satisfactory. The inevitable drawback, owing to coincidences with the ζ', ξ, ι, κ, λ, and other not yet computed series, is of no great importance, because of the small intensities of these series. If this number of isolated lines had been uniformly distributed, the

exact computation of all constants would have been an easy matter. But while the interval between 5154 and 5130 abounds in isolated lines, very useful for the special investigations of the series of the first head, the other parts of the band are rather poor in isolated lines.

Only in the case of isolated lines can we properly indicate the precision of the measures by their mean errors. Thus only by means of this interval 5154–5130 an approximate calculation of the mean error of a single observation by Kayser and Runge to $\pm$ 0.020 has been possible by means of our catalogue of differences (Obs.—Comp.). This value accords very well with the statements of the observers themselves. Under the most favorable circumstances indeed their measures are so accurate that they deserved to be given with three decimal places instead of two. It is not only in the hope of very accurate future observations that I have given my tables and comparisons with a greater number of decimal places, but also in respect to the measures of Messrs. Kayser and Runge.

In the cases of feebler and indistinct lines, no doubt the mean errors are greater, and as Kayser and Runge very seldom have resolved duplicities with less than 0.1 difference in wavelength, we can understand that faint lines with supposed differences of 0.15, or even more from strong ones, may easily have been overlooked. I confess, however, that not a few such lines, which I have found it more convenient to identify with Kayser and Runge lines, perhaps ought to have been mentioned as *hp* lines in my catalogue.

COPENHAGEN OBSERVATORY,
April 1898.

THE SPECTRA AND PROPER MOTION OF STARS.

By W. H. S. Monck.

In January 1893 I contributed to *Astronomy and Astro-Physics* an article on the spectra and proper motion of stars, and the same number contained one by Mr. Maclair Boraston dealing, at least partially, with the same subject. Five years have naturally led to further researches on the subject, and the views which I then expressed need some modification, though I think they were substantially correct.

In the first place the use of more powerful spectroscopes has led Professor Pickering to alter somewhat the classification of spectra which he adopted in the *Draper Catalogue*. The types of spectra which he now adopts as regards the vast majority of the stars are A, B, F, G, K and M, with intermediate spectra designated in some such manner as A5F or F2G. He agrees with the view which I then expressed in placing the type B above A in respect of brilliancy, so that the first two letters of the alphabet are transposed in his list. I divided these stars (except those of the type M) into three classes, Sirian, Capellan, and Arcturian: but I left out the stars described as G in the *Draper Catalogue* because I was doubtful as to which of the two latter classes they belonged to. There is now no doubt that they are Capellans, for most of the stars classed as E in the *Draper Catalogue* become G in a more powerful spectroscope. This of course would make some changes in my figures; and I also obtained, partly from published sources and partly from private communications, for which I have to thank Professor Pickering, a number of corrections in the *Draper Catalogue*. I was likewise able to use a fuller catalogue of stars with large proper motion than I believe had hitherto been available—I mean that of M. Bossert of the Paris Observatory, which (including the appendix) contains 2675 stars, all of which have proper motions of not less

than one-tenth of a second per annum in one or other of the elements.

The small proper motion of the Sirian stars compared with those of other types being known and acknowledged, I did not think it necessary to ascertain the corrected number of Sirians in the *Draper Catalogue*. It cannot be less than 5000. But it was otherwise with those of the Capellan and Arcturian types, because it was necessary to ascertain whether the preponderance of the former among the stars of large proper motion was or was not due to their greater frequency in the sky. My count may not be perfectly accurate even on the information now attainable, but I think it is so substantially. I made the Arcturian stars 2386 against 2293 Capellans. I think I may safely say that the latter are not more numerous than the former, and I have little doubt that they are less so. I was not, however, led to Mr. Boraston's conclusion that these two classes of stars are distributed in a similar manner over the sky. Following the order of right ascension as in the *Draper Catalogue*, the first hour showed a preponderance of Arcturians, but then there was a decided preponderance of Capellans until about the 9^h 30^m, up to which point they outnumbered the Arcturians by about 160. Then there was another change, and from this to the 24th hour the Arcturian majority was about 250. There were several pages of the catalogue in which the stars of one class were more than twice as numerous as those of the other. I did not, however, attempt to trace the laws of distribution more minutely.

With this preface I give the results of my comparison of M. Bossert's catalogue with the *Draper Catalogue*, utilizing also a few spectra of Southern stars that have been published by Professor Pickering. The result is :

Sirian stars, - - - - - -	225
Capellan stars, - - - - - -	461
Arcturian stars, - - - - - -	366

Intermediate types I reckoned as ½ to each, merely omitting the fraction at the end of the result (strictly, I made 225½ Sirian). The Capellan stars with large proper motion thus

exceed the Arcturian in the proportion of fully five to four, while they are probably less numerous on the whole. They exceed the Sirians in the proportion of more than two to one, while they are less than half as numerous on the whole.

Stars of the type M seem to possess considerable proper motion according to this comparison. There are ninety-eight of them in the *Draper Catalogue* and twenty in that of Bossert. The number, however, is rather too small for a final decision, especially as the M is marked with a query in a large proportion of the cases. Stars with peculiar spectra — not reducible to any of the foregoing heads — seem, generally speaking, to have very small proper motions.

As to the figures which I have given, it will be noted that M. Bossert includes in his catalogue stars with less proper motion than any other catalogue of stars with *large* proper motion that I had hitherto examined. Of course the lower we descend the larger will be the proportion of Sirian stars and the more nearly will the Capellans and Arcturians become equal, or rather tend to exhibit a slight excess of the latter. Had M. Bossert stopped at one-fifth instead of one-tenth of a second, the distinction between the three classes would have been more strongly marked.

Though somewhat doubtful as to the positions of stars of the types G and M, my present arrangement in order of brilliancy (and consequently of average distance for stars of the same magnitude) would be B, A, K, M, F, G. If this order is correct, it seems clear that no continuous gradation of spectra can be traced through it. Arcturian stars are not Capellans which have cooled down to a lower stage. If they were so, then proper motions would, on the average, be greater, not less than those of the uncooled Capellans. Cooling would reduce the light of the star without affecting its proper motion. Consequently the cooled star would, on the average, have the greater proper motion, the magnitudes (*i. e.* quantity of light) being supposed equal. Are the Capellan stars cooled-down Sirians? The difference in the amount of their average proper motions is start-

ling. But then Professor Pickering has, on fuller examination, discovered many intermediate types, and as far as I am at present able to form an opinion, these intermediate types exhibit intermediate proper motions. I do not think we are as yet in a position to form any confident opinion. But an examination of binary stars with reliable orbits confirms my conclusion that the Arcturian stars are not cooled-down Capellans and that on the supposition that these two types represent different stages of star-life, the Capellan, not the Arcturian stars must represent the later stage.

ON THE RELATIVE INTENSITIES OF THE LINES IN THE SPECTRUM OF THE ORION NEBULA.

By C. RUNGE.

ON a visit to the Lick Observatory in September 1897 I was allowed, through the kindness of Professor Campbell, to confirm his observations of the differences in the relative intensities of the lines in different parts of the Orion nebula (see *Astronomische Nachrichten*, No. 3471). To these observations Professor Scheiner raises the objection that the differences are not real, but merely physiological effects due to the peculiar constitution of the human eye (*A. N.*, No. 3476). He very rightly insists that these physiological effects ought to be taken into account in a discussion of observations of this kind. But, as I believe I am able to show, he overrates the physiological effect in assuming that it suffices to explain the whole observed differences.

The physiological effect (the so-called Purkinje phenomenon) is this: Two objects emitting light of different colors do not maintain the same apparent relation of intensities when their real intensities are altered in equal proportions. A bright red object, for instance, of apparently the same brightness as a blue object, appears very much darker than the latter on reducing the real intensities of both in equal proportions. To keep the apparent intensities equal, the light of shorter wave-length must be reduced very much more than the light of longer wave-length. This reduction may be determined in quantity and thus furnishes a measure of the physiological effect. A. König has investigated the subject very thoroughly.[1] If we call B_λ and $B_{\lambda'}$ the apparent intensities of two objects at medium brightness emitting light of wave-lengths λ and λ', and if we call b_λ and $b_{\lambda'}$ the apparent intensities of the same objects at low bright-

[1] A. KÖNIG, "Abhängigkeit der Farben und Helligkeitsgleichungen von der absoluten Intensität," *Sitzungsber. d. K. Akad. d. W., Berlin*, July 1897. See also, "Helligkeitswerth der Spektralfarben," *Festschrift zum 70. Geburtstage von H. von Helmholtz*, by the same author.

ness after reducing the real intensities in equal proportions, then the value of $\frac{b_\lambda}{b_{\lambda'}} : \frac{B_\lambda}{B_{\lambda'}}$ measures the effect in question. This value, as A. König finds, does not practically change when the medium brightness or the low brightness is changed within considerable limits, always supposing that the real intensities of both colors keep the same relative intensity. It is also independent of the relation of the real intensities. With the color comparing instrument devised by Helmholtz and improved by A. König two semicircular fields of about 4° apparent magnitude can be made to emit light of any two wave-lengths. The intensity of each field can be altered separately by turning one of a pair of Nicols, and the intensity of both fields can be altered simultaneously in the same proportion by turning one of a pair of Nicols in the eyepiece. The angle between the planes of polarization of the two Nicols determines the amount of reduction of real intensity, the real intensity being proportional to the square of the cosine.

Through the kindness of Professor A. König I was allowed the use of his instrument to measure the Purkinje effect with the colors of the three nebular lines 4861, 4959, 5007. Comparing $\lambda = 4861$ and $\lambda' = 5007$ I first regulated the intensities so that both fields appeared to be of equal medium brightness ($B_\lambda = B_{\lambda'}$). By turning the Nicol in the eyepiece both intensities were reduced in the same proportion to low brightness (about $\frac{1}{200}$ of the medium intensity). The field $\lambda = 4861$ then appeared decidedly brighter than the other and was reduced to equal apparent intensity by turning a Nicol through a certain angle. The two positions of the Nicol were read. Then the Nicol in the eyepiece was turned back and the brightness again increased from low to medium. The fields equal at low brightness appeared unequal at medium brightness, the shorter wave-length this time giving the weaker field. The intensity of this field was then again increased to equal apparent intensity by turning the Nicol. The two positions of the Nicol were read. We may shortly state the proceedings thus:

(1) $B_\lambda = B_{\lambda'}$, $b_\lambda > b_{\lambda'}$, the proportion $\frac{b_\lambda}{b_{\lambda'}}$ was measured.

(2) $b_\lambda = b_{\lambda'}$, $B_{\lambda'} > B_\lambda$, the proportion $\frac{B_{\lambda'}}{B_\lambda}$ was measured.

Two sets of five settings each gave for $B_\lambda = B_{\lambda'}$, $\frac{b_\lambda}{b_{\lambda'}} = 1.81$ and 1.74 (mean $= 1.78$), and two other sets of five settings each gave for $b_\lambda = b_{\lambda'}$, $\frac{B_{\lambda'}}{B_\lambda} = 1.78$ and 1.73 (mean $= 1.76$). Both cases are contained in the equation

$$\frac{b_\lambda}{b_{\lambda'}} : \frac{B_\lambda}{B_{\lambda'}} = 1.8$$

or

$$\frac{b_\lambda}{b_{\lambda'}} = 1.8\,\frac{B_\lambda}{B_{\lambda'}}.$$

I may therefore say that on reducing the brightness from a medium to a low degree the wave-length 4861 gained on 5007 so that the relative intensity was nearly doubled. Professor A. König, who had the kindness to repeat the measurements for me, did not find the effect so strong for his eye. His determinations were:

$$(1)\qquad B_\lambda = B_{\lambda'},\ \frac{b_\lambda}{b_{\lambda'}} = 1.57 \text{ and } 1.38$$

$$(2)\qquad b_\lambda = b_{\lambda'},\ \frac{B_{\lambda'}}{B_\lambda} = 1.54 \text{ and } 1.47$$

$$\text{mean } \frac{b_\lambda}{b_{\lambda'}} = 1.49\,\frac{B_\lambda}{B_{\lambda'}}$$

The effect seems to decrease when the image of the field is formed on parts of the retina away from the fovea.

According to these results we should indeed expect the nebular lines 4861 and 5007 to alter their apparent relative intensity in more or less bright parts of the nebula, even if there were no real alteration of the relative intensity. $\frac{B_{4861}}{B_{5007}}$ might, according to my measurements, apparently increase in weaker parts of the nebula, but it would not increase more than 1.8 times, unless there was a real increase of relative intensity.

Now I estimated $\frac{B_{4861}}{B_{5007}}$ in brighter parts of the nebula as between $\frac{1}{3}$ and $\frac{2}{5}$ and in a weaker part of the nebula as equal to 10. The increase was therefore between twenty-five and thirty times. I do not place much confidence in the exact values of my estimations; but I have no doubt that a change in relative intensity, which I estimated to be in the proportion 1 : 25, cannot have been as small as 1 : 2.

In order to see if my estimation of the relative intensity 2 : 5 and 10 : 1 somewhat agreed with the measurements of the instrument, I tried by estimation to set the two fields in the colors $\lambda = 4861$ and 5007 to these relative intensities and afterwards measured them. The estimated relative intensity 2 : 5 I found 2 : 4, and the estimated relative intensity 10 : 1 I found 8.5 : 1. However, I did not satisfy myself how far this agreement was due to chance.

Another proof that the differences in the relative intensities of the nebular lines really exist, is furnished by comparing the lines 4861 and 4959. Near the trapezium they appear equally bright. In the neighborhood of Bond 734 the line 4861 is well discernible (10 times brighter than 5007), while 4959 could no longer be seen. Now I have made one field in the color-comparing apparatus emit light of wave-length 4861 and the other light of wave-length 4959 of apparently equal brightness. On lowering the intensities of both fields by turning the nicol of the eyepiece the Purkinje effect was plainly visible, 4861 appearing a little brighter. On lowering the intensity still more, so that the two fields were just visible, the effect was lost, and then the two fields vanished simultaneously. On increasing the intensity they again appeared simultaneously. This is not reconcilable with the observations of the two nebular lines 4861 and 4959, unless there is a real difference in the relative intensities. The proof is less objectionable than the first, as no estimation of relative intensities enters into it.

Professor Scheiner has observed the Purkinje effect with the colors of $H\alpha$ and $H\beta$. That is altogether a different thing.

According to A. König's measurements[1] the relative apparent intensity $\frac{H\beta}{H\alpha}$ increases about 1000 times on reducing the real intensities in equal proportions from medium to low brightness. The Purkinje effect is therefore about 600 times stronger than with the colors $H\beta$ and 5007. I agree with Professor Scheiner that the apparent absence of $H\alpha$ in the spectrum of many nebulæ may be due to this purely physiological cause.

One might increase the evidence of real differences in the relative intensities of the nebular lines. I do not think that it would give any serious difficulty to measure the relative intensities of the nebular lines instead of estimating them. And then instead of measuring the Purkinje effect as I have done with semicircular fields, it would be safer to measure it directly with the lines of the brighter portion of the nebula and to compare the difference of relative intensities with the difference observed on shifting the slit to other parts of the nebula.

Technische Hochschule,
Hannover, May 1898.

[1] *Sitzungsber. d. K. Akad. d. W., Berlin*, July 1897, p. 881.

THE ECHELON SPECTROSCOPE.[1]

By A. A. Michelson.

The resolving power of a diffraction grating is proportional to the product of the total number of lines by the order of the spectrum observed. But little effort seems to have been made to make a decided step in the direction of increasing the *order* of the spectrum observed, and this is doubtless because for a grating acting by *opacity* the brightness of the spectrum diminishes very rapidly as the order increases. This difficulty has been successfully overcome by ruling the lines in such a way as to concentrate the greater proportion of light in one spectrum, but so far as I am aware such attempts have been limited to the first, second or third spectrum and the results even here are somewhat fortuitous.

It seems nevertheless quite possible to construct gratings which shall throw a quite large proportion of the light in very high orders of spectra — say the hundredth — in which case the grating space must be of the order of a hundred waves or say twenty to the millimeter, instead of a thousand. The lines would have to be drawn with no more accuracy than before, and the grating could be completed in a very short time and temperature changes would have a much smaller effect than at present.

It may be that there are more serious practical difficulties in the way of such a ruling as is represented in Fig. 1 than would be anticipated. Especially may this be true if the greater part of the light is to be returned in the direction from which it came; for that the grooves must be correspondingly deep, and the grating space would vary with the depth.[2] Fig. 1 at once suggests a possible method of effecting the same result, by

[1] The first part of this paper was published as a preliminary notice in the *Am. Jour. Sci.* for March.

[2] This question will receive a practical test as soon as a ruling machine now under construction is completed.

building up the steps by equal thicknesses of optical glass. Here the difficulty, even supposing the optical work to be

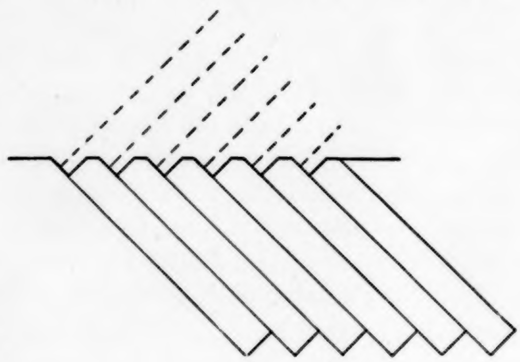

FIG. 1.

practically perfect, would be the joining of the separate plates in such a way as to have always the same distance between them.

By using the same arrangement for transmission instead of reflection this difficulty is avoided — and there remains absolutely nothing but the difficulty of making a considerable number of plane-parallel plates of the same thickness — to an order of accuracy only one-fourth that required in the former arrangement, or even one-tenth of this if the other medium be water or oil instead of air.

Probably the surprising thing is the smallness of the number of plates required to give results which are comparable with those of the best gratings. This can be shown as follows: Let abd (Fig. 2) be one step in the series of plates and let $ab = s$ and $bd = t$. If m is the order of the spectrum observed, $m\lambda = \mu \,.\, bd - ac$ or

$$m\lambda = \mu t - t\cos\theta + s\sin\theta$$

$$\frac{d\theta}{d\lambda} = \frac{m - t\dfrac{d\mu}{d\lambda}}{t\sin\theta + s\cos\theta}$$

$$\frac{d\theta_1}{dm} = \frac{\lambda}{t\sin\theta + s\cos\theta}$$

and if $\delta\theta$ is the displacement corresponding to $\delta\lambda$ and $\delta\theta_1$ that corresponding to $\delta m = 1$, assuming Cauchy's formula

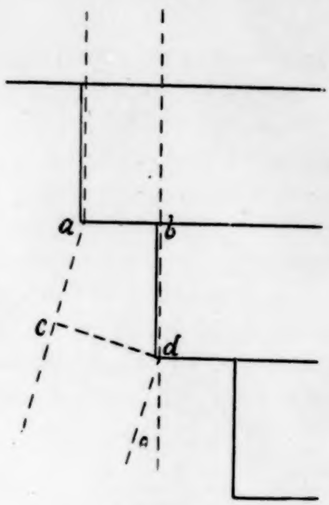

FIG. 2.

$\mu = a + b / \lambda^2$, and taking the approximate value of $m = (\mu - 1)\, t / \lambda$, we have

$$\delta\theta / \delta\theta_1 = [\,(\mu - 1) + 2\,(\mu - a)\,]\, \frac{t}{\lambda} \cdot \frac{\delta\lambda}{\lambda}.$$

For flint glass the coefficient of t / λ is approximately equal to unity; so that if $\frac{\delta\lambda}{\lambda} = .001$, as in the case of the two sodium lines, and $t = 5^{mm} = 10000\,\lambda$, then $d\theta = 10\ d\theta_1$, that is, the sodium lines would be separated by ten times the distance between the spectra.

The resolving power of such a combination is mn, exactly as in the case of gratings; so that with but *twenty* elements 5^{mm} thick and hence $m = 5000$ the resolving power would be 100000, which is about that of the best gratings.

The experiment was actually tried with but *seven* elements, placed between a collimator and an observing telescope and the collimator slit illuminated with light from a sodium flame.

The images were so distinct that the broadening of the lines could be very easily detected, and the Zeeman effect was readily observed when the sodium flame was placed in a magnetic field.

It is important to note that the resolving power is independent of the number of plates but depends only on the total thickness, and the only advantage in a large number of elements is the greater separation of the spectra. The overlapping of spectra is doubtless a disadvantage, which however could be overcome by a preliminary analysis; and for the examination of single lines and especially in the investigation of effects of broadening, shifting, or doubling of lines, the method seems especially well adapted.

An echelon spectroscope of twenty elements, the essential parts of which were constructed in the Ryerson Physical Laboratory, is represented in Fig. 3.

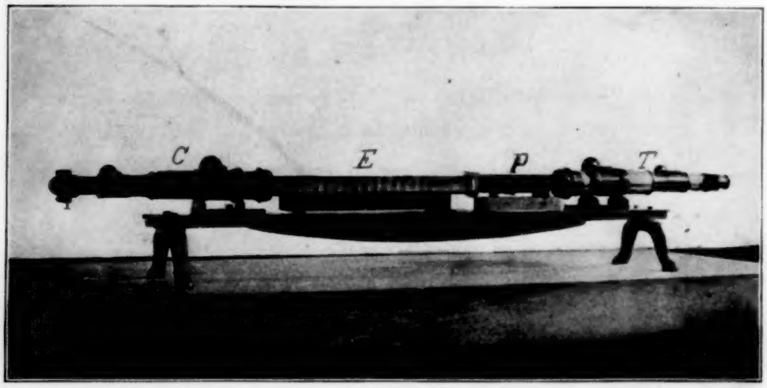

FIG. 3.

C is the collimator and *T* the observing telescope; *E* the echelon consisting of twenty plates, each 18^{mm} thick, and diminishing in width from 22^{mm} to 2^{mm}, so that the width of the elementary pencils is 1^{mm}, and the successive retardations are of the order of twenty thousand waves.

From what precedes it follows that practically all the light may be concentrated in one spectrum, so that the only losses are those due to the reflections and absorptions, and these are much less than in either gratings or prisms of equal resolving power. The intensity is given by the formula

$$I = \left[\frac{\sin \pi \frac{s}{\lambda} \theta}{\pi \frac{s}{\lambda} \theta} \right]^2$$

or by the curve Fig. 4.

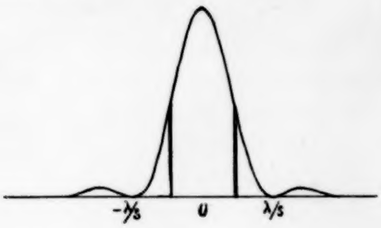

FIG. 4.

All the light is practically included between the deviations $-\frac{\lambda}{s}$ and $\frac{\lambda}{s}$. But the distance between successive spectra is $\frac{\lambda}{s}$, so that in general there will be *two* spectra visible. By slightly inclining the echelon however, the directions of the spectra may be varied so that one falls at O, where the intensity is a maximum, while the adjoining ones disappear.

The theoretical resolving power of the twenty element echelon is 15000 × 20 or 300000; and the results which are given below show that this limit is nearly reached in practice.

The overlapping of the spectra is overcome by a direct vision prism of moderate dispersion, but the distance between the spectra is so small in comparison with the dispersion of the echelon that the spectrum of the source under examination must consist of rather fine lines if overlapping is to be avoided. Thus, in the present case the range of wave-length corresponding to two successive spectra is $\frac{\delta\lambda}{\lambda} = \frac{2\lambda}{t} = \frac{1}{15000}$; that is,

the lines to be examined must have a total width of less than one-fifteenth of the distance between the sodium lines.

This caused some difficulty in the examination of certain phenomena, as for instance, the Zeeman effect, and it will doubtless be an improvement to have three times the number of elements of one-third the thickness — for then with equal resolving power, the range would be three times as great.

The increased number of the plates will of course increase the losses by reflections, and the degree of accuracy in working the plates must also be correspondingly increased. Thus for twenty plates the retardations must differ by less than one-twentieth of a wave, while with sixty the difference must be less than a sixtieth of a wave.

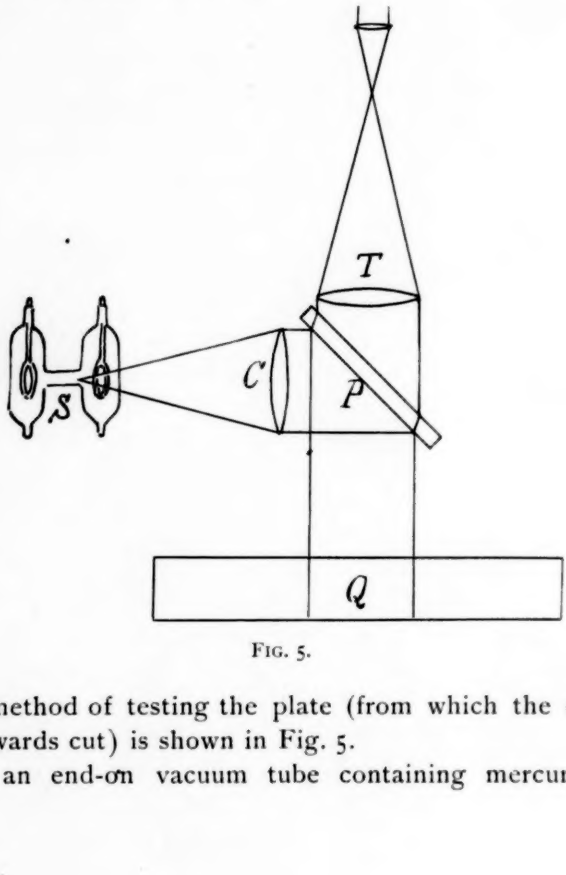

FIG. 5.

The method of testing the plate (from which the elements are afterwards cut) is shown in Fig. 5.

S is an end-on vacuum tube containing mercury vapor

which is illuminated by the spark of an induction coil. The green radiation is so much brighter than the others that it is not necessary to use a spectroscope. The light, after being brought to an approximately parallel beam by the collimator C, is reflected from the lightly silvered plate P to the plate Q to be tested. From both surfaces of Q it is returned normally, and part passing through P, is examined by a low-power telescope T.

When the two surfaces of Q are nearly parallel, circular interference fringes begin to appear, which become more clearly defined as the accuracy of parallelism increases. At this stage the plate Q is moved so that all parts of the surface are examined in turn, and any residual errors of parallelism are at once detected by a contraction or expansion of the circles. The surface is locally corrected till these disappear. Under ordinary circumstances it is quite easy to distinguish a difference in phase of one-twentieth, which would correspond to a difference of retardation of the transmitted light of one-eightieth of a wave, so that there would be no serious difficulty in constructing an echelon of eighty or even a hundred elements.

In order to test the practical efficiency of the instrument, a somewhat extended investigation of the Zeeman effect was undertaken.

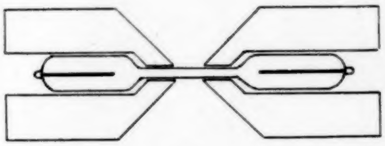

FIG. 6.

As has been already indicated in a previous paper,[1] in order to observe the complete analysis of the radiations in a magnetic field, it is necessary that these should be as nearly homogeneous as possible, and this is the case only when the radiations take place under low pressure, so that the substance must be placed in a vacuum tube from which the air is exhausted to 5–10 ^{mm} residual pressure, and illuminated by the electric dis-

[1] *Phil. Mag.*, September 1892.

charge. If the discharge takes place in a direction perpendicular to the field there is a lateral displacement of the discharge current, which interferes seriously with the result when the field is strong. In order to avoid this difficulty the pole pieces were arranged as in Fig. 6.

In the case of non-volatile substances the self-induction spark, between terminals of the substance to be examined, was used. The arrangement is shown in Fig. 7.

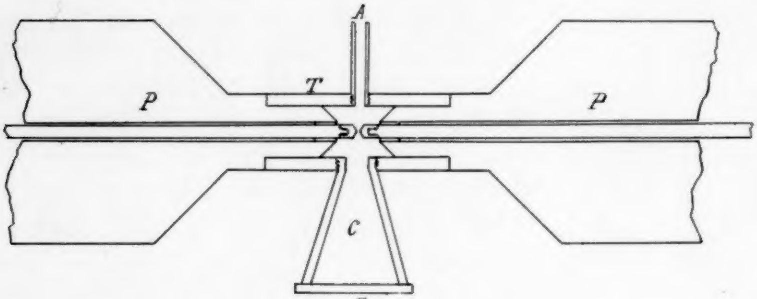

FIG. 7.

The pole-pieces are bored axially to receive the rods *r r*, to the ends of which the terminals are screwed.[1] The brass tube *T* is fitted air-tight by melted beeswax to the pole-pieces, and communicates by the tube *A* with a mercury pump, and a glass plate *G* closes the conical tube *C*. One of the rods *r* is fitted with a screw so that the corresponding terminal may be adjusted midway between the pole-pieces. The other rod is given a rapid oscillatory motion by a small electric motor, the joint being closed by a rubber tube.[2]

Under these circumstances a spark of considerable intensity is produced at every break of contact between the terminals with a storage battery of ten cells. The beeswax joints permit

[1] By making the rods and one of the terminals of iron, the field may be increased several times.

[2] This arrangement was only partially successful. The spark in the magnetic field was frequently too faint to observe, though occasionally the reverse was true.

an exhaustion to within a few millimeters of mercury, which may be maintained as long as necessary.

The observations completely confirm the experiments made by the method of visibility curves.[1] In particular the distribution for the cadmium lines is as represented in Fig. 8.

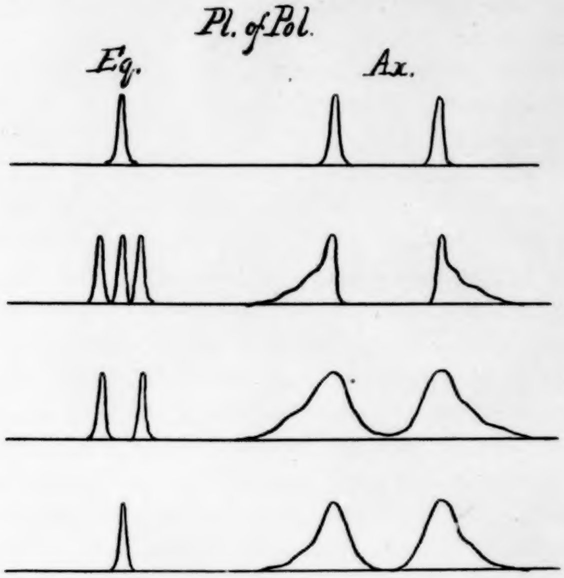

FIG. 8.

In addition to the lines previously classified[2] the following are added:

Gold, yellow line, class II.
Gold, green line, class I.
Silver, yellow line, class I.
Silver, green line, class I.
Copper, yellow line, class IV.
Copper, green lines, class I.

[1] This JOURNAL, February 1898. The "outer lines" of the Zeeman triplet show indications of complex structure, but were actually resolved in the case of the green mercury line only; the difficulty arising from the overlapping of adjacent spectra when the field was sufficiently powerful to separate the elements of the group.

[2] *Ibid.*

Magnesium, green line (5183), class III.
Magnesium, green line (5172), class II.
Magnesium, green line (5167), class I.
Manganese, green (5340), class IV.
Argon, red line, class I.
Tin, red line (6450), class II.
Tin, yellow line (5798), class I.
Tin, yellow line (5587), class I.
Tin, yellow line (5564), class I.
Iron, yellow lines, class I.
Iron, blue lines, class I.

The central line of the iron triplets are all broadened, especially the blue lines.

A very remarkable effect is observed in the case of the yellow copper line. The line without the field is a close double, the distance being one-one hundred and fiftieth of the distance between the D lines, or 0.04 A. U. As the field increases the lines merge together without broadening, and with a strong field there is but a single very narrow line.

The behavior of the yellow-green line of manganese is even more striking. The line is a quadruple line, just resolvable. In a weak magnetic field the light accumulates at the center of the group, the lines becoming indistinct and merging together. In a strong field the quadruple band is reduced to a single fine line at the center of the group.

These two cases are the only ones of this character thus far observed, though doubtless a systematic search will reveal many others. They may be considered as a new type of line to be added to the three classes previously described.

Another interesting case is that of sodium. Without the field each component is a close double,[1] the distribution being that represented in Fig. 9*a*.

With a strong magnetic field and plane of polarization equatorial, the distribution is represented in Fig. 9*b*.

In *a* the elements of D_2 are about two-thirds as far apart as D_1, while in *b* the reverse is the case.

[1] This had been pointed out before. See *Phil. Mag.*, September 1892.

Finally, the distance between the components in *a* is a function of the density of the sodium vapor, increasing from zero for low densities to about one-sixtieth of D_1–D_2 for the greatest

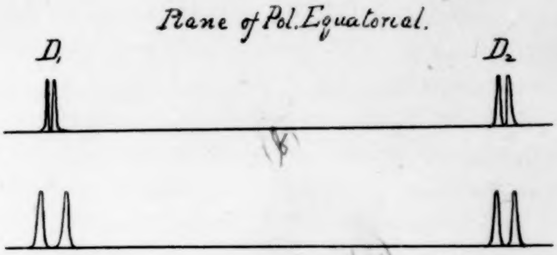

FIG. 9.

density obtained without broadening, and finally obliterating the lines.

For the investigation of very close lines the present instrument has not quite sufficient resolving power. For instance, it has been shown[1] that the green mercury line is a complex system whose principal components are less than 0.01 A. U. apart, while the theoretical limit of resolution of the echelon used is about twice as great.

A new echelon is now under construction which will have a resolving power about five times as great as the present instrument, and this will amply suffice for the analysis of the closes groups that are likely to occur.

[1] *Phil. Mag.*, September 1892.

NOTES ON THE ZEEMAN EFFECT.

By J. S. Ames, R. F. Earhart and H. M. Reese.

In verifying the Zeeman Effect, so-called, or the modification of radiations in the ether produced by the magnetic field, we have observed certain variations from the phenomena as originally described which seem worthy of note. Up to the present time we have studied the spark-spectra of iron, cadmium, zinc, and magnesium. The investigation is by no means completed; but it may be worth while to give a brief account of our work, and to defer until later a fuller discussion.

The apparatus used was the small concave grating spectroscope of the Physical Laboratory of the Johns Hopkins University, which has a radius of curvature of about twelve feet. The grating is ruled with 15,000 lines to the inch, and is five inches in width. The magnetic field was produced by a common type of electro-magnet, but we made no attempt to measure the intensity of the field, because our object was not to establish numerical relations. The field, however, was strong enough to produce a separation in the case of the "magnetic triplet" of over one-tenth of an Ångström unit. Our method of studying the effect was to place the spark gap between the poles of the magnet, introduce between the spark and the slit a Nicol's prism and a quartz lens, to photograph the resulting spectra along the middle of the photographic plate, having the Nicol's prism placed with its principal plane perpendicular to the field of force, then turning the Nicol's prism through 90° and at the same time rotating a horizontal shutter which is placed in front of the photographic plate, to thus expose the two edges of the plate to the new radiation coming through the Nicol's prism. By this method we secure on the same plate the components of the vibrations polarized along the lines of force and those polarized at right angles to them.

In the case of iron, we have studied the spectrum from wave-

length 3500 to wave-length 4400, and have made a careful investigation of all the lines in this region. With certain exceptions presently to be noted, all the lines are influenced in the way discovered by Zeeman. In particular, when the radiation at right angles to the magnetic field is studied, each line in the spectrum is in general broken up into three, the central component being plane polarized with its vibrations along the lines of force, the two side components being plane polarized at right angles to these, their vibrations being at right angles to the field of force.

We have observed, however, that three lines, of wave-length 3587.13, 3733.47, and 3865.67, are affected in the opposite way; that is, the line is a triplet when viewed at right angles to the magnetic field, but the central component is so polarized that its vibrations are at right angles to the field, and the two side components have their vibrations along the field. Two lines, one at 3722.72, the other at 3872.64, are so modified as to be quadruplets, the central component, which has its vibrations along the lines of force, being a close double. Several lines, among which are those at 3746.06, 3767.34, 3850.12, and 3888.67, have, so far as our indications go, no modification produced whatever. There are several lines concerning which we have doubt, but the majority of the other lines are certainly modified in the way described by Zeeman. (Several of the above phenomena have been observed by other investigators, notably Deslandres and Cornu.)

We have observed, also, that the separation of the side components of the triplets seemed to be irregular. In studying this, however, we noticed that there were certain lines in which the separation was nearly the same, but much greater than that of other lines whose separations seemed to be quite closely alike; that is, the lines of the iron spectrum, as studied, seemed to break up into two classes in each of which the "magnetic separation" is the same, but in the one set much greater than in the other. On tabulating these lines belonging to the two sets it was at once observed that there was a striking agreement with

the sets into which the iron spectrum breaks up when studied with reference to the shift produced by pressure.

In the article by Dr. W. J. Humphreys in this JOURNAL, October 1897, there is given a list of eighteen lines in the spectrum of iron which have a moderately small shift, whereas seven have an abnormally large shift. We found, with no exception, that those lines which had a great magnetic shift were those which had a great pressure shift. (Our attention was called in this investigation to a slight error in Dr. Humphrey's paper which he was able to correct at once from his laboratory note-book. The line at 4236.112 should be erased from those lines which have a small shift, and among the lines which have a large shift the one at wave-length 4271.934 should read 4271.325.)

This agreement between the separation or change produced by a magnetic field and by pressure direct is most interesting and will undoubtedly lead to a better understanding of both phenomena.

In studying the spectrum of cadmium we noticed that all lines belonging to the second subsidiary series which we could investigate had magnetic separations nearly the same but much greater than the magnetic separation produced in those lines of the first subordinate series which we could study. This action is perfectly in accord with the phenomenon noted above in the case of iron.

Our investigation of the spectra of zinc and magnesium is not yet complete enough to warrant us in making any statement.

JOHNS HOPKINS UNIVERSITY,
May 1898.

THE STRUCTURE OF THE SHADING OF THE H AND K AND SOME OTHER LINES IN THE SPECTRUM OF THE SUN AND ARC.

By L. E. Jewell.

Several years ago, while examining a series of photographs of the solar spectrum, made by Professor Rowland in 1888 and 1889, I discovered one plate on which the shading of the H and K lines (due to calcium vapor) was broken up into bands or series. In each case the bands began at the center of the shaded lines and extended outward, the distance apart of the component lines of the series increasing as the distance from the center increased. In each case the series were perfectly symmetrical about the centers of H and K, and the individual lines or components of the series somewhat nebulous, while nearly all other small lines in the same region were sharp, clear lines. All other plates available were carefully examined without the presence of series being indicated. Possibly a very few of the stronger lines in the series may be present upon most plates, but these lines may be merely approximately coincident, and not connected with the series. A great many plates of this region of the spectrum have been taken by myself during the last few years, over the center of the Sun, at the Sun's limb, over Sun-spots, and under various conditions ; but the presence of these series has never been certainly indicated. It is probable that the principal reason for this is that all the negatives taken during several years past have been upon commercial plates, which are coarse grained, thus masking the faint lines of the series, while the plate upon which they are shown is almost grainless.

There are several plates taken in the third order spectrum which show the shading of some of the strongest of the iron lines broken up into series ; but the component lines are faint, nebulous, and close together. There is also suspicion of the partial resolution of the shading of the strongest lines of some

other elements, but it is less certain. All plates taken of the arc-spectrum of the calcium lines H, K, and *g* ($\lambda = 3933.809$, 3968.620, and 4226.892), have been carefully examined without any result until the 11th of last March, when a plate taken under somewhat special conditions showed the shading broken up into series.

I had for a number of years suspected that the shading of lines was produced by the overlapping of many series all converging towards the principal line, but that under ordinary circumstances it is impossible to separate any series from the others (a possible exception occurs in the shaded lines of tin, which are plainly resolved into series).

It was thought probable that the distance apart of the lines in any particular series depended upon the degree of crowding of the molecules or density of the gas producing the shaded lines, and consequently the damping of the original vibration.

In both the electric arc and the Sun's atmosphere we probably have layers of metallic vapors of varying degrees of density, so that the combination will generally produce a smooth shading, gradually increasing in intensity towards the central line.

In the case of the solar spectrum plate under consideration, it was thought that the slit of the spectroscope probably covered a region of the Sun's atmosphere where the principal layer of calcium gas was of a particular density; and, being thus to a large degree isolated, it was able to produce its characteristic series. This idea is somewhat confirmed by the fact that the general shading of H and K on the plate is unusually weak. These particular conditions would probably be most likely to occur over faculæ.

On March 11, having occasion to photograph the arc-spectrum of calcium at λ 4000, I took several plates under widely different conditions. One plate shows the shading of H on the red side quite distinctly broken up into series similar to those of the solar spectrum plate mentioned. The resolution into series on the violet side is less distinct, while the shading of K shows the resolution better upon the violet than upon the

red side, though not so distinctly as upon the red side of H. The resolution of the shading of *g* into series is uncertain. If the lines of the series are present they are so nebulous and indistinct as to not be easily recognized. The resolution into lines is hardly perceptible close to the principal line, but is fairly distinct about three Ångström units from H. In the solar spectrum plate the lines of the series are much more distinct nearer the central line (about 1 or 2 Ångström units away) and become too faint to follow at any great distance. They are much more distinct each side of K than around H.

In the arc-spectrum plate it is also to be noted that the lines of the series are reversed or absorption lines instead of emission lines, although at some distance away from the central lines it is probable that the series are continued as faint emission lines.

The arc-spectrum plate showing these series was obtained by using an extremely powerful direct electric current, allowing it to act for a short time before throwing the image of the arc upon the slit of the spectroscope, and then exposing for three or four seconds only. Under these conditions the calcium was rapidly volatilized and the highly heated vapor formed a much more extended atmosphere around the poles than with a weaker current, and it is also possible that the conditions throughout the larger part of the arc were more uniform than under ordinary circumstances.

JOHNS HOPKINS UNIVERSITY,
May 25, 1898.

Minor Contributions and Notes.

". . . . TEACH ME HOW TO NAME THE . . . . LIGHT."

It would be a convenience if a name were chosen for the as yet undiscovered gas, which is suggested by the typical bright nebular lines, as a principal constituent of the nebulæ. Sir William Huggins has used occasionally the term *nebulum*. Independently, Miss Agnes Clerke has made the suggestion to me of *nebulium* as an appropriate term, which, "though not unobjectionable from an etymological point of view, is on all fours with *coronium*." If, however, the Greek nomenclature adopted for *helium* and *argon* is to be followed, the term *nephelium* or *nephium* may be suggested as suitable;—for, probably, *asterium* would be thought too general in its meaning. It is most desirable that the name chosen should be one universally acceptable to astrophysicists, and so exclusively adopted. Hence, this note.

Margaret L. Huggins.

ASTRONOMICAL AND PHYSICAL CONFERENCE AT THE HARVARD COLLEGE OBSERVATORY.

As the conference held at the Yerkes Observatory in October 1897 was successful in bringing together a large number of astronomers and physicists, whose contributions in the form of papers and discussions rendered the proceedings of great interest and value, it was felt that an attempt should be made to provide for a repetition of the conference in the summer of 1898, with a view to its continuance in future years. A preliminary inquiry, directed to those who attended the previous conference, and to certain others who were considered likely to accept, brought a large number of satisfactory replies. The writers were practically unanimous in favoring a continuation of the conferences, and definite acceptances were so numerous as to insure the success of a second conference at the Yerkes Observatory. It was found, however, that the geographical position of the Observatory would prevent many persons in the eastern states from coming. No practical way of continuing the meetings and at the same time avoiding this

difficulty presented itself until Professor Pickering offered to hold the 1898 conference at the Harvard College Observatory, beginning on Thursday, August 18, and continuing until the following Saturday. The advantages of this plan are so obvious that a large attendance may confidently be expected. On Monday, August 21, the Fiftieth Anniversary Meeting of the American Association for the Advancement of Science opens in Boston, and members of the conference can thus be present on this important occasion. By joining the American Association those who are not already members can take advantage of the low rates offered by the railroads. It is hoped that the attendance of the conference may greatly exceed that of last October, and that the meetings may be so successful as to warrant their repetition in future years.

G. E. H.

VARIABLE STARS OF SHORT PERIOD.[1]

WHOEVER will make a careful examination of the brightness of a large number of stars either in the sky, or better, as photographed upon different plates, will be impressed with the vast number which show no perceptible variation. The discovery of variable stars is greatly aided when we are able to make a suitable selection for examination, either from their spectra or from their presence in clusters. Visually, we can never be sure that all the variables in a given region have been found, however carefully we may study them. Photography brings this problem more nearly within our reach, and a partial solution of it is illustrated in the accompanying figure. A photographic telescope was constructed having as an objective a Cooke Anastigmatic Lens, with an aperture of $2^{cm}.6$, and a focal length of $33^{cm}.3$. This telescope was mounted equatorially and the lens was alternately exposed and covered for intervals of exactly 10 and 50 minutes by an electrical attachment. The polar axis of the mounting was displaced and the rate of the driving clock was increased, so that the successive images should be slightly separated. An 8×10 photographic plate was exposed in this instrument on April 21, 1898, and eight successive images were obtained, the Greenwich Mean Times of the middle of the exposures being $13^h\ 49^m$, $14^h\ 49^m$, $15^h\ 49^m$, $16^h\ 49^m$, $17^h\ 49^m$, $18^h\ 48^m$, $19^h\ 48^m$, and $20^h\ 48^m$. The plate covered a region

[1] *Harvard College Observatory Circular* No. 29.

about 33° square, whose center was R. A. $= 1^h\ 2^m$, Dec. $= +76°.6$. The images of the stars in the corners of the plate were sufficiently good when visible to show very slight variations in light, but owing to their increased size the faintest stars were not shown. The greatest

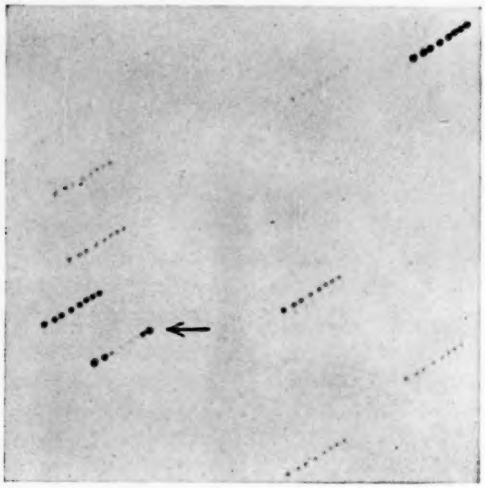

FIG. 1.

loss amounted to about one magnitude. If now any variable star having a period of less than fourteen hours was contained in this region it is probable that at least one maximum and one minimum would be photographed. The figure represents a portion of the plate described above, enlarged ten times to a scale of $60'' = 0^{cm}.1$, and covers about one square degree. It therefore represents one-thousandth of the entire plate, the size of which on this scale would be two meters, or nearly seven feet square. The entire sky, from the north to the south pole, could be covered by forty such plates, and it is proposed to do this as soon as the best method of taking the plates has been determined. The arrow indicates the variable star U Cephei, and its photometric magnitudes at the times the eight images were taken were 7.5, 8.1, 8.9, 9.1, 9.1, 8.3, 7.6, and 7.2. The three stars above it are +81°30, +81°27, and +81°29, which have the photometric magnitudes 7.9, 8.5, and 8.6. To separate the successive images various methods have been tried. The best of these seems to be stopping the

driving clock for a few seconds every hour. By the above plan we hope to secure a complete list of all variable stars of short period brighter than the ninth magnitude at maximum whose variation exceeds half a magnitude, and whose period is less than a day. Doubtless many other variable stars of longer period, and stars of the Algol type may also be incidentally found.

EDWARD C. PICKERING.

MAY 21, 1898.

THE SUPPOSED VARIABLE STAR, Y AQUILAE.[1]

MEASURES to determine the light curves of variable stars of short period, north of declination -40°, are now in progress with the meridian photometer. Four sets of four settings each are ordinarily made when the star to be observed is about half an hour east of the meridian, and again about an hour later. These measures are repeated on twenty or thirty nights. The principal error is that due to the unequal transparency of the air in different portions of the sky, the stars compared being often far apart. The accidental errors of measurement are small, owing to the number of settings. Smooth light curves have been found for all the stars thus measured, with the exception of $+10^\circ 3787$. The designation Y Aquilae was given to this star by Mr. S. C. Chandler, and in his catalogue of variable stars he states that it varies from magnitude 5.3 to 5.7 in a period of 4.986 days. Also, that it was "Suspected by Gould, confirmed by Chandler, 1894; also by Yendell." It will be noticed that the period is so nearly five days that for several months the same phase will recur at about the same hour angle, thus permitting errors to occur in visual observations by Argelander's method, such as are mentioned in *Circular* No. 23, and which led to such wholly erroneous conclusions in the case of U Pegasi.

The star $+10^\circ$ 3787 was observed with the meridian photometer on nineteen nights, from August 25 to October 13, 1897. Placing together the observations having the same phase, we find, corresponding to the phases $0^d.0$, $1^d.0$, $2^d.0$, $3^d.0$, and $4^d.0$, the mean residuals -0.09, 0.00, $+0.02$, $+0.02$, and $+0.03$. We might infer a variation with a range of a tenth of a magnitude, but the first value, -0.09, depends on observations on a single night, and the range of the other four is only 0.03. This star is No. 39, of the standards selected by

[1] *Harvard College Observatory Circular* No. 30.

the observers at Potsdam, and they state that from their observations they find no confirmation of the variation suspected by Dr. Gould. Mr. Chandler, however (*Astron. Jour.*, **14**, 135), states that these observations, fifty-seven in number, confirm the period of variation that he has found, although indicating that the range of variation is less. A reduction of these observations made here, however, leads to the same conclusion as that found at Potsdam. The actual number of observations of this star made at Potsdam appears to be seventy five, not fifty-seven, forty-nine of them in determining the light of the standards, and twenty-six in the supplementary zones. Grouping them according to phase, we obtain the residuals —0.07, —0.02, +0.01, 0.00, and —0.16, corresponding to the phases $0^d.6$, $1^d.6$, $2^d.6$, $3^d.6$, and $4^d.6$. The last residual is reduced to —0.12 if we reject the observations on one night, and to —0.09, if we reject those made on two nights, and use only the observations from which the light of the fundamental stars was determined. To decide whether such residuals as these are due to accident, the Potsdam observations were again grouped, assuming a period of six days instead of five. The mean residuals corresponding to the phases 0, 1, 2, 3, 4, and 5 days then become —0.03, —0.19, —0.14, —0.02, +0.01, and 0.00. The preponderance of negative residuals is caused by the supplementary observations, which indicate that the star was slightly brighter than when the standards were originally measured. Grouping the observations with the meridian photometer in the same way, we obtain the mean residuals 0.00, 0.00, —0.07, —0.02, —0.02, and +0.07. In both cases, therefore, a period taken at random indicates variability more clearly than that hitherto assumed. Mr. Yendell (*Astron. Jour.*, **14**, 160), confirms visually the variability of this star.

The accuracy of the observations described in *Circulars* Nos. 23 and 25, seems to afford a conclusive test of the variability of this star. Accordingly, comparisons were made by Mr. Yendell with the photometer attached to the 15-inch equatorial telescope on May 9, 10, 13, 14, 21, and 22, 1898. Eighty settings were made each night. The comparison star was $+10^\circ$ 3784. The mean differences of magnitude were 3.63, 3.64, 3.66, 3.66, 3.66, and 3.62, and the average deviations from their means, of the five groups of sixteen settings each on the different nights, were ±0.022, ±0.028, ±0.020, ±0.012, ±0.030, and ±0.012. The first and fourth nights have nearly the same phase, $0^d.8$. Combining these in one group, and placing the others in the order of

phase, we have the phases, $0^d.8$, $1^d.8$, $2^d.8$, $3^d.9$, and $4^d.8$, and the corresponding residuals, 0.00, 0.00, —0.02, +0.02, and —0.02. A positive residual, as usual, denotes that the star was faint. The average value of these residuals is $\pm$0.012, and the range 0.04.

The three series of photometric observations discussed above, therefore, fail to show any evidence of variation, since deviations of a tenth of a magnitude, except in the last series of measures, may be ascribed to errors of observation. Since it is impossible to prove that the light of a star never changes, this star may still be an Algol variable with a short time of variation, or the period may be entirely wrong.

EDWARD C. PICKERING.

MAY 25, 1898.

ELECTION OF EDWIN BRANT FROST AS PROFESSOR OF ASTROPHYSICS AT THE YERKES OBSERVATORY.

THE staff of the Yerkes Observatory will soon be materially strengthened by the addition of Edwin Brant Frost, Director of the Shattuck Observatory of Dartmouth College, who has been elected Professor of Astrophysics in the University of Chicago. A gift of $15,000 to the University has been recently made by Miss Catherine W. Bruce for the special purpose of providing for this appointment. Professor Frost expects to devote special attention to a study of stellar spectra with the forty-inch telescope. He will also continue to assist in the editorial work of the ASTROPHYSICAL JOURNAL.

G. E. H.

Reviews.

Sur une nouvelle Méthode de Spectroscopie Interferentielle. A. PEROT et CH. FABRY, *Comptes Rendus*, t. CXXVI, p. 34.

Étude de quelques Radiations par la Spectroscopie Interferentielle. A. PEROT et CH. FABRY, *Comptes Rendus*, t. CXXVI, p. 407.

THE interferential spectroscope described in these two papers consists of two plane parallel glasses enclosing a film of air between them. Suitable mountings allow the distance between the glasses, and consequently the thickness of the air film to be varied from nearly zero to any desired amount.

When illuminated by a beam of monochromatic light the film of air produces the usual interference phenomena of thin plates. Such plates give rise to two complementary sets of interference fringes, the reflected and the transmitted, the latter being the one utilized by Perot and Fabry.

Airy's well-known formula for the transmitted system is

$$I = \frac{a^2\,(1-e^2)^2}{(1-e^2)^2 + 4\,e^2 \sin^2\left(\frac{\pi}{\lambda}\mathrm{V}\right)},$$

where I is the illumination, a^2 the intensity of the incident light, and e the reflection coefficient of the surfaces bounding the film. The intensity of a bright fringe is then a^2, while that of a dark one is

$$\frac{a^2\,(1-e^2)^2}{(1+e^2)^2},$$

and their ratio is consequently

$$\frac{(1+e^2)^2}{(1-e^2)^2}.$$

In case the film is air and the bounding surfaces glass, at normal incidence $e = 0.2$, and we obtain as the ratio of the intensity of a bright fringe to that of a dark band

$$\left(\frac{1+.04}{1-.04}\right)^2 = 1.18.$$

The contrast would then be small and the fringes would be lost in the general illumination. If, however, the glasses bounding the air film are lightly silvered on their inner surfaces, e may become as great as 0.87 and we get for the ratio 49. In this case the fringes are very prominent, the dark ones being nearly black. It is under these conditions that the fringes have been employed in the interferential spectroscope.

The incident beam is made slightly convergent and the transmitted system of fringes is viewed through a telescope focused for nearly parallel rays. The surfaces of the glasses being parallel to each other, with a convergent beam the fringes are concentric circles, the common center being the normal to the film passing through the axis of the observing telescope.

In Airy's formula the incident light is considered to be strictly monochromatic. It is easy to see, however, that if the incident beam contains light of various wave-lengths, each kind of light will give rise to an independent system of fringes, which may or may not coincide. For instance if the incident beam consists of light of two very nearly equal wave-lengths, there will be two systems of fringes, which will coincide when the distance between the glasses is small, and hence will appear to the observer as a single system. When, however, by a gradual separation of the glasses a considerable difference of path is attained, the coincidence will no longer exist and both sets of fringes will be visible.

In order to measure the relative wave-lengths of the two kinds of light, the difference of path is so adjusted that the bright fringes of one system coincide exactly with the dark ones of the other system. If p is the number of the fringe at which this takes place, λ and $\lambda + \Delta\lambda$ the wave-lengths of the incident light, and e the distance between the surfaces, we evidently have

$$(p + \tfrac{1}{2})\,\lambda = p\,(\lambda + \Delta\lambda) = 2\,e,$$

whence

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{2p} = \frac{\lambda + \Delta\lambda}{4e}.$$

Therefore by observing the order of the fringe at which the bright fringes of one system coincide in position with the dark ones of the other system we obtain the value of $\dfrac{\Delta\lambda}{\lambda}$.

The authors show that in favorable cases, *i. e.*, those in which the component lines are very narrow, two lines whose distance apart is only $\frac{1}{1000}$ of that between the D_1 and the D_2 lines may be separated.

Perot and Fabry have applied their instrument to the study of certain radiations which have previously been analyzed by Michelson with the interferometer. Their results are in some cases quite different from those of Michelson; for instance, where Michelson finds the green thallium line ($\lambda = 5349$) to consist of four lines, Perot and Fabry find it to be a triplet, the distances between the components agreeing however with the values given by Michelson.

In the case of the blue cadmium line ($\lambda = 4800$) which Michelson decides to be a double, Perot and Fabry find it to be a triplet, consisting of a principal radiation with a companion on each side at distances of 0.08 Ångström units.

In order to explain these differences Perot and Fabry call attention to the fact that in the visibility curve of Michelson, which is given by $V^2 = \frac{C^2 + S^2}{P^2}$, C and S are indeterminate, and can thus be chosen in any manner whatever so as to satisfy the equation of the visibility curve. In order to determine them uniquely a second condition is necessary.

While this statement is perfectly true in the most general case, yet in those cases where the distribution is symmetrical the terms expressed by S vanish, $V = \frac{C}{P}$, and the criticism does not apply. It has been shown that in the radiations thus far examined, the distribution of light in the ultimate lines agrees with Maxwell's formula for the velocity distribution in a gas and is therefore symmetrical.

In order to gain an idea as to the effectiveness of the new instrument it seems most natural to compare its performance with that of the interferometer.

The interferential spectroscope has the advantage of showing directly the structure of a given radiation by a simple inspection of the system of fringes at considerable differences of path, and therefore does not depend upon a series of estimates of visibilities, as is the case with the interferometer. Each fringe is in fact a true spectrum of the source, and the conditions are precisely the same as those existing in the spectra obtained by the use of a grating possessing a small number of lines, but with which spectra of a very high order can be observed.

Furthermore, in order to show that a given radiation is complex it is not necessary to attain such great differences of path as are necessary in the interferometer.

From the visibility curve obtained with the interferometer it is impossible to say whether a companion line is situated on the red or on the violet side of the principal line, while with the interferential spectroscope this is at once revealed by the way in which the doubling takes place.

The visibility curve, however, allows the determination of the breadth of each component line with a considerable degree of accuracy, while the interferential spectroscope as used by its inventors does not. This is quite an important advantage that the interferometer possesses, and it will no doubt be recalled that measurements of this sort have given us probably the most direct proof of the kinetic theory of gases.

Another advantage possessed by the interferometer is that the difference of path may be made zero, and yet the distance between the semi-silvered glass and the two mirrors is considerable, thus rendering the instrument very suitable for investigations concerning refractive indices, etc.

In conclusion it may be said that Messrs. Perot and Fabry deserve the thanks of physicists for placing in their hands an instrument of such power and efficiency, and if the problem of the mechanism of radiation is ever solved it seems that the new instrument will take an important part in the solution.

A. St. C. D.

NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation,

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed. If a request is sent *with the manuscript* one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

The Editors do not hold themselves responsible for opinions expressed by contributors.

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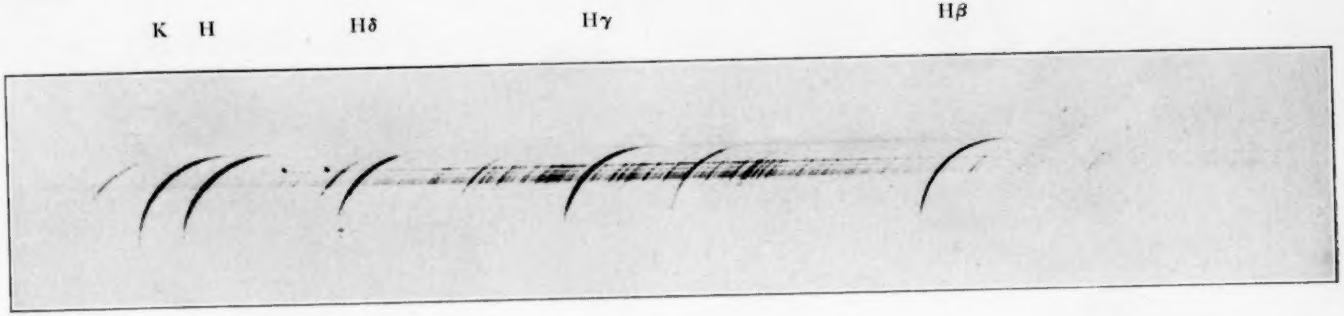
PLATE I.



SPECTRUM OF THE "FLASH" PHOTOGRAPHED BY PROFESSOR K. D. NAEGAMVALA
AT THE ECLIPSE OF JANUARY 21, 1898.